

# Production Networks and the Flattening of the Phillips Curve\*

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## Abstract

This paper analyzes the role of changes in the structure of production networks on the flattening of the Phillips curve over the last decades. I build a multi-sector model with production networks, and heterogeneity in input-output linkages and in degree of nominal rigidities. In the production network model, inflation sensitivity to the output gap depends on the topology of the network of the economy. In particular, I show that two characteristics of the network matter for inflation dynamics: (i) the network multiplier and (ii) output shares. Analyzing the U.S. Input-Output structure from 1963 to 2017, I document structural changes in the production network. Calibrating the model to these sectoral changes can account for a decrease in the slope of up to 15 percent. Decomposing the aggregate effect shows that the flattening is primarily due to an increase in the centrality of sectors with more rigid prices that is incompletely reflected by compositional changes in value-added.

**JEL codes:** C67, E23, E31, E32, E52, E58

**Key words:** Production Networks, Inflation Dynamics, Phillips Curve, Sectoral Heterogeneity, Nominal Rigidities, New-Keynesian Model

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# 1 Introduction

The strength of the relationship between inflation and economic activity, represented by the Phillips curve, has been at the centerstage of discussions between economic commentators and policymakers in the past few years. Empirical studies have found a flattening of the slope of the Phillips curve over time. This is of central importance to policymakers and central banks in particular because the sensitivity of inflation to the output gap is important for many reasons. It gives a sense of how real activity affects inflation. For instance, given a positive output gap, a smaller sensitivity implies smaller inflationary pressures. In this situation, maintaining an inflation target will become harder for a central bank. In order to reach the same target level, larger movements in economic activity are needed, which in turn require larger shifts in the interest rate. This is of particular concern, in times of the zero lower bound on the interest rate.

The evidence on the flattening documents that the sensitivity of inflation to output has declined by more than 50 percent, with most of the change taking place in a period after the 1980's.<sup>1</sup> Understanding the sources of this shift is crucial and economists have suggested a number of possible explanations. Commonly proposed explanations include the success of monetary policy in anchoring expectations (Bernanke, 2010), the credibility of the central bank (McLeay and Tenreyro, 2019), or global forces (Jordá et al., 2019). Those explanations have different implications for how optimal policy would need to change: from a larger role of fiscal policies or combined money-fiscal policies (Gali, 2019) towards rethinking inflation targeting.

In this paper, I propose a novel explanation for the flattening of the Phillips curve. I investigate the implications of changes to the production network structure of the economy for inflation dynamics. These changes go beyond changes in the value-added composition of the economy. Networks are important since firms use a variety of inputs to build their products. Thereby, they form a complex web of input-output linkages. Analysis of the input-output tables of the U.S. economy show large changes in those interlinkages that coincide with the changes in inflation in the 1980's. Changes in the input-output structure have implications for the sensitivity of inflation as they alter sectoral input-output linkages. I show how the slope of the Phillips curve depends on the topology of the network. Moreover, using historical data on the input-output linkages, I find that a network-augmented Phillips curve can account for a part of the flattening of the Phillips curve since the mid 1980's.

Inflation dynamics depend on the network structure of the economy. In this paper, I study a multi-sector economy with monetary frictions in which industries are connected through input-output linkages. Additionally, I consider heterogeneity across the network structure, the degree of nominal rigidities and markups. The first main result of this paper is that two key network

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<sup>1</sup>See for instance Kiley (2000), Ball and Mazumder (2011), Blanchard, Cerutti and Summers (2015) Coibin and Gorodnichenko (2015), or for a recent overview Stock and Watson (2019). Studies on the wage Phillips curve include Gali and Gambetti (2019) or Hooper, Mishkin and Sufi (2019).

statistics matter for inflation dynamics: (i) *network multiplier* and (ii) *centrality captured by sectoral gross output shares*. These network statistics describe specific attributes of the input-output linkages, based on fundamentals of the economy. They have direct empirical counterparts that can be easily observed.

The network multiplier is a measure of the overall importance of the network in this economy. Total production in an economy exceeds real value-added (GDP) by the intermediate good use. The larger intermediate good use, the stronger fluctuations in GDP will impact total production and therefore labor demand and prices.

The output share is a measure of network centrality. A sector's output share captures the importance of the sector's output to all other sectors as an input as well as for the consumption good. If the output of a sector is extensively used by other sectors in the economy, then its equilibrium output share will be high. Whether a sector has a high or low output share depends on the network structure. Sectors with larger output shares will contribute more to the input prices of other sectors and therefore to aggregate inflation dynamics. If central sectors have higher degrees of nominal rigidities, the aggregate sensitivity of inflation in this economy will be smaller.

The importance of a sector in the economy will not only be given by its value added-share but rather by its gross output share. A standard multi-sector model predicts that the importance of sector and therefore the extent towards it affects aggregate inflation is related to the share of that sectors in final consumption. In the production network model, instead, a sector can have a positive influence on the aggregate inflation dynamics even if its consumption share is zero.

The network structure affects aggregate inflation dynamics through another channel that dampens the sensitivity of inflation: *strategic complementarities*. When the optimal price chosen by a firm depends positively (negatively) on the prices of other firms, we speak of strategic complementarities (substitutes) (see Cooper and John, 1988). Here, strategic complementarities arise because of sticky intermediate good prices. In my production network setting, however, there are two key differences with strategic complementarities in standard formulations of intermediate goods as in Basu (1995). First, prices depend positively on the sector-specific input price instead of the aggregate price level. The sectoral input price depends on the composition of the sectoral input good, which in turn depends on the composition of those goods constituting inputs. As an implication, the degree of strategic complementarity depends on the particular network structure of the whole economy because of those indirect supply channels. Second, the degree of strategic complementarity will be sector-specific and be larger for sectors that have a larger share of intermediate goods used in production.

A second implication concerns the estimation of the Phillips curve. Inflation dynamics are determined by endogenous variables in addition to the output gap. The presence of these variables biases the estimated slope coefficient of the standard Phillips curve because they are correlated

with the output gap. As I demonstrate, the bias depends on the network structure. Therefore, the evolution of the Phillips curve could either be caused by a decrease in the standard slope coefficient or by a change in the bias through changes in the endogenous variables. I show that additionally to the former effect, changes in the network structure influence the correlation between these endogenous variables and the output gap, which leads to lower estimates of the Phillips curve.

The network structure of the U.S. economy has changed over time. The Bureau of Economic Analysis (BEA) provides Input-Output accounts for the U.S. economy from which a snapshot of the production economy of the U.S. economy can be drawn. Figures 1 and 2 provide network representations of the input-output linkages, in which nodes (circles) represent sectors and edges (lines) represent input flows between sectors the color of the node shows the originating sector. The color of nodes captures whether a sector belongs to one of three broad categories: (i) manufacturing (blue), (ii) services (red) (iii) and (iii) others (orange). Furthermore, the size of a node represents the sectors centrality as measured by its output share, and the thickness of an edge corresponds to the importance of the node. Centrality of manufacturing firms as well as selected other sectors such as construction or farms, captured by the size of the blue and orange nodes, has decreased between those two periods. Conversely, the centrality of service increased as reflected by the size of red nodes in 2017. This process of reallocation of centrality illustrates the structural changes in the production network, while normally, structural transformation refers to the change in the value-added (GDP) shares of sectors.<sup>2</sup>

I study the role of the structural changes in the production network on the flattening of the Phillips curve by fitting the multi-sector production network model to the input-output structures for each year between 1963 and 2017. For each period, I then estimate the model implied sensitivity of inflation to the output gap. The baseline calibration of the model shows a flattening of the Phillips curve that is consistent with empirical evidence on the shape and timing of the flattening. While before 1980 and after 2000, the slope shows a diverging behavior in the data and the model, there is a clear flattening in the 80's and 90's. From the peak in the 80's until the beginning of the 21st century, the slope of the calibrated model decreases by about 15%.

The most important channel contributing to this evolution is that changes in the production network have shifted centrality towards sectors with higher nominal rigidities. This is equivalent to an increase in the aggregate degree of nominal rigidity in the economy. Specifically, aggregate inflation has become more rigid because services' prices are much stickier than manufacturing' prices. There is evidence from micro studies showing that service prices are more rigid than those in the manufacturing sector, e.g. Bils and Klenow (2004), Klenow and Kryvtsov (2008)

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<sup>2</sup>Rachedi and Galesi (2016) document an increase in the service share of intermediate inputs and refer to this process as services deepening.

or Nakamura and Steinsson (2008). Increases in the degree of nominal rigidity translate into a smaller sensitivity of inflation to the output gap in the Phillips curve.

Taking into account the input-output structure of the economy is important to understand the decline in the Phillips curve. Simple compositional changes in value-added fail to capture all of the explained changes to the Phillips curve. The increase in aggregate rigidity due to sectoral reallocation exceeds the one that would arise considering changes in value-added shares only. Using the model I can decompose the aggregate change to the slope estimate into the contribution of each of those two channels. I find that changes in the network structure and in the value-added shares each contribute half to the explained decline in the slope of the Phillips curve.

This paper relates to the literature on sectoral heterogeneity and production networks. It is connected to studies on the implications of networks on the transmission of shocks (e.g. Horvath, 2000, Acemoglu et al. 2012, 2015 or Carvalho, 2014). In these studies, the size of the network effects in the amplification of shocks is usually related to the Leontief-Inverse matrix (Acemoglu et al., 2015, or Bigio and La’O, 2019). I contribute two new insights to this literature. First, I show that the impact of the network on the transmission of shocks depends on two network statistics that capture different components of the network effects: (i) the importance of the overall network and (ii) the relative importance of sectors. Second, in the presence of nominal frictions, the network statistics and hence the size of network effects become dependent on countercyclical markups.

The paper is also related to New Keynesian models with production networks. It is connected to studies that emphasize the role of networks and sectoral heterogeneity in price rigidity in amplifying the degree of aggregate monetary non-neutrality (e.g. Carvalho, 2006, Galesi and Rachedi, 2019, and Pasten, Schoenle and Weber, 2019a), on government spending multipliers (Bouakez, Rachedi and Santoro, 2019), or the role of price dispersion on optimal policy (Cienfuegos, 2019). My paper also ties in closely with Pasten, Schoenle and Weber (2019b), who argue that in the presence of heterogeneity in intermediate input consumption and nominal rigidities the relevant measure of the size of a sector changes.<sup>3</sup> In contrast to these studies, I focus on the role of production networks (and changes to it) on inflation dynamics. Moreover, I depart by allowing for a more general network structure via heterogeneity in sectoral intermediate good shares and sectoral degrees of market power. I discuss the implications of this model for the slope of the Phillips curve, and calibrate it for the U.S. economy at different points in time to compare the estimated slopes of the Phillips curve.

Section 2 of this paper presents evidence of the evolution of the Phillips curve. Section 3 outlines the structure of the model and explains the importance of the two network statistics.

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<sup>3</sup>In particular, the effective distribution of size and centrality (out-degree) argument resembles my distinction between output shares and value-added shares.

Section 4 describes the calibration of the model and shows how the network structure as measured by the two network statistics has changed over time. Section 5 investigates inflation dynamics and predictions of the model for the sensitivity of inflation to the output gap by comparing different network economies. Section 6 shows the solution of the model over time and reports the implied slope of the Phillips curve. Finally, Section 7 concludes.

## 2 Evidence on the Flattening of the Phillips Curve

Before turning to networks as a potential explanation for changes in the Phillips curve, in this section, I present evidence on the evolution of the Phillips curve over the past 50 years. I will focus on three components: (i) size of the change, (ii) shape of the change and (iii) timing of the change.

### 2.1 Coefficient over Time

At the center of macroeconomics is the theory that the real and nominal side of the economy are linked through a Phillips curve relationship. Phillips (1958) provided the first formal statistical evidence on this trade-off using data on wage inflation in the U.K. Samuelson and Solow (1960) extended “Phillips’ curve” to U.S. data and to price inflation. In this paper, I focus on the most widely used model of this kind, the New Keynesian Phillips curve (NKPC). It gained popularity from its theoretical microfoundations that build on early work of Fischer (1997), Taylor (1980) and Calvo (1983). It is centered around staggered price setting by forward looking individuals and firms.<sup>4</sup> The key property of the NKPC is that inflation dynamics reflect changes in economic activity and inflationary expectations. The standard macroeconomic textbook version of the NKPC as in Woodford (2003) or Galí (2015) is

$$\pi_t = c + \beta E_t \pi_{t+1} + \kappa x_t + v_t \tag{1}$$

where  $c$  is a constant,  $x$  is a measure of economic activity,  $\beta$  the time discount factor,  $v$  corresponds to cost-push shocks and  $E_t \pi_{t+1}$  denotes expectations of inflation. The measure of economic activity in these models is usually marginal costs which in turn is related to the output gap. The coefficient  $\kappa$  here describes the relationship between economic slack and inflationary pressures, i.e. the slope of the Phillips curve.<sup>5</sup>

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<sup>4</sup>Achieved by two standard ingredients: a microeconomic environment with (i) monopolistically competitive firms (ii) facing constraints on price adjustment.

<sup>5</sup>In these models, the slope is usually given by  $\kappa = (1 - \theta)(1 - \theta\beta)/\theta * (\sigma + \varphi)$  where  $\theta$  is the Calvo parameter - probability of not adjusting prices,  $\beta$  corresponds to the time discount factor,  $\sigma$  denotes the intertemporal elasticity of substitution and  $\varphi$  is the Frisch labor supply elasticity.

I continue by estimating this relationship for the U.S. economy between 1960 and 2007.<sup>6</sup> To characterize the strength of economic activity, I use estimates of the Congressional Budget Office for the potential level of GDP. Concerning inflation expectations, I follow Ball and Mazumder (2011) as well as Coibin and Gorodnichenko (2015), and assume as a simple baseline that inflation expectations are backward-looking. Specifically, I assume that inflation expectations are a four-quarter average of past inflation rates,

$$\mathbb{E}_t \pi_{t+1} = \frac{1}{4}(\pi_{t-1} + \pi_{t-2} + \pi_{t-3} + \pi_{t-4}).$$

where I use the inflation rate from the personal consumption expenditure survey (PCE),  $\pi_t$ .

First, I investigate how the sensitivity of inflationary dynamics to economic activity has changed over time. Was there a particular point in time when the slope broke down or was this rather a smooth process? Has the slope only flattened or was there a time when it was increasing? To answer these questions, I estimate the relationship (1) by OLS over rolling windows of 50 quarters.

In Figure 3, I report the average relationship between the output gap and deviations of inflation from expectations for each window,  $\kappa$ , together with the one standard deviation confidence intervals. Two results stand out. First, the slope of the Phillips curve has not always been flattening. Instead, we can basically observe three episodes since 1975. In the first part of the sample and up the middle of the 80's, we can observe an increasing slope of the Phillips curve. In the second period which goes until the start of the 21 century, there is an apparent flattening of the slope. In the final phase, the relationship diverges on a low level, with periods in which the average relationship is significant and insignificant.

Second, there has not been a particular event that reduced the slope permanently. Instead, we can observe a protracted episode in which the slope has decreased since the mid 80's. Therefore, the results of this section indicate that the flattening of the Phillips curve is a smooth process that started in the middle of the 1980's.

## 2.2 Timing of the Flattening

After identifying the shape of the flattening of the Phillips curve, I investigate the exact timing of the change. Therefore, I employ a Andrews (1963) test for parameter instability with unknown break date. I investigate the statistical evidence for a structural break in the relationship between inflation dynamics and the output gap by formally allowing for a break in the relationship at unknown  $\tau$  as follows:

$$\pi_t - \beta E_t \pi_{t+1} = c + \kappa_1 * I(t < \tau) * x_t + \kappa_2 * I(t \geq \tau) * x_t + v_t$$

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<sup>6</sup>The online appendix shows that these estimates are robust to different specifications, for instance concerning the measure of inflation, or the output gap.

where  $I$  are time dummy variables equal to one if the respective condition is satisfied and zero otherwise. The null hypothesis of the Andrews test is that the slope coefficients in both periods are equal.

Table 7, reports results for the Andrews test for different specifications for the inflation measure, forcing variable and estimation methodology. Consistent with results of the previous section, the test cannot reject the null that the slope is unchanged. The protracted flattening episode is represented by a series of potential break dates in 1982 or 1983.

Next, I will use the previous result on the break date to calculate the size of the change in the slope coefficient. In Figure 4, I present a scatter plot of quarterly output gaps for the United States against the deviations of inflation that quarter from expected, discounted inflation. Data from 1960Q1 until our estimated break date 1982Q3 is represented by circles, while data from 1982Q4 is plotted by diamonds. The lines represent the slope of the average relationship estimated by OLS over each sample period. The slope is positive, indicating that economic slack, i.e. economic activity that is lower than potential, is associated with inflationary pressures below expectations. The sensitivity has changed over time. We can observe a flattening in the sensitivity of inflation to economic activity and estimates of the slope suggest that those are in the magnitude of 40 to 80 percent.

The statistical evidence of this section provided insights into the size, timing and shape of the flattening of the Phillips curve. The findings suggest that the flattening was a smooth process that started in 1982Q2 and saw a decrease in the slope of 40-80%.

### 3 Model

I consider a multi-sector New Keynesian Model with nominal rigidities and linkages in production via the use of sector-specific intermediate goods. In comparison to standard New Keynesian (e.g. Gali, 2015) or multi-sector models, firms use as inputs to production not only labor but also goods produced by firms from potentially all sectors of the economy. Additionally, heterogeneity in the degree of nominal rigidities, elasticity of substitution, and intermediate good share are modeled. The model represents an extension of the standard New Keynesian model (Gali, 2015), with the sectoral models of Carvalho (2006), Carvalho and Lee (2011) and Pasten, Schoenle and Weber (2019), as well as the intermediate good model of Basu (1995) as limiting cases.<sup>7</sup> The economy consists of firms, households, and a government.

The economy is composed of a continuum of firms  $i \in [0, 1]$  each of which belongs to a sector  $k \in \{1, 2, \dots, K\}$ . Each firm produces a differentiated good that can be used either in

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<sup>7</sup>Carvalho (2006) considers a multi-sector model with heterogeneity in nominal rigidities. Carvalho and Lee (2011) and Pasten, Schoenle and Weber (2019) add roundabout production structures. Cienfuegos (2018) studies a model with trend inflation and without heterogeneity in the elasticity of substitution or and intermediate good shares.



consumption or in the production of other goods. Within each sector firms face monopolistic competition (Dixit-Stiglitz), and produce with the same Cobb-Douglas production function that combines labor and intermediate inputs (with share  $\gamma_k$ ). They set prices à la Calvo (1983), i.e. they can reset prices with an exogenous, but sector-specific probability,  $\theta_k$ .

On the consumption side, the economy is represented by a single representative household that chooses labor and aggregate consumption. The latter is composed of sectoral consumption bundles, which themselves are CES aggregators of goods produced by individual firms within a sector. The labor aggregator is CES, too.

The government consists of a monetary authority, which sets the nominal interest rate following a Taylor rule.

### 3.1 Households

The economy is populated by an infinitely-lived representative household that has preferences over consumption,  $C_t$  and labor,  $L_t$ . She seeks to maximize expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right] \quad (2)$$

where  $C_t$  is aggregate (final) consumption,  $L_t$  is labor input,  $\beta$  is the subjective time discount factor,  $\sigma$  is intertemporal elasticity of substitution,  $\varphi$  inverse of the Frisch elasticity of labor supply and  $E_0$  is the expectations operator conditional on information up to time  $t = 0$ .

The aggregate consumption bundle is a CD aggregator of sectoral consumption bundles

$$C_t = \prod_{k=1}^K C_{k,t}^{\vartheta_k} \quad (3)$$

where  $\vartheta_k$  is the expenditure share of sectoral consumption from sector  $k$ . Also  $\sum_k \vartheta_k = 1$ . The sectoral consumption bundles  $C_{k,t}$  themselves are CES aggregators of the individual firms indexed in  $[0, 1]$

$$C_{k,t} = \left[ \int_0^1 C_{k,t}(i)^{\frac{\epsilon_k-1}{\epsilon_k}} \right]^{\frac{\epsilon_k}{\epsilon_k-1}} \quad (4)$$

where  $C_{k,t}(i)$  is the quantity of good  $i$  of sector  $k$  consumed by the household, and  $\epsilon_k$  is the sector-specific elasticity of substitution between different goods of a sector. It is a measure of competitiveness in sectors. Note also that one usually assumes  $\epsilon_k > 1$  which implies that it is harder to substitute consumption goods from different sectors than substitute goods within the same sector.

I assume that labor provided by the household to the firms cannot move perfectly across sectors. Lee and Wolpin (2006) document that there are large mobility costs that impairs the

sectoral allocation of labor. To capture this feature, I follow Horvath (2000) and Kim and Kim (2006) and model the aggregate labor bundle as a CES aggregator of sectoral labor supply,  $L_{k,t}$ , that is

$$L_t = \left[ \sum_{k=1}^K L_{k,t}^{(1+\nu)/\nu} \right]^{\nu/(1+\nu)} \quad (5)$$

where  $\nu$  gives labor mobility.<sup>8</sup> At  $\nu = \infty$ , labor is perfectly mobile, and all sectors pay the same wage.

The household purchases a bundle of consumption goods and allocates the remaining income to the purchase of new bonds. She derives income from providing labor, receiving nominal profits from firms and interest on her bond holdings. The period budget constraint is therefore given by

$$\sum_{k=1}^K \int_0^1 P_{k,t}(i) C_{k,t}(i) di + B_t = \sum_k W_{k,t} L_{k,t} + I_{t-1} B_t + \sum_{k=1}^K \int_0^1 \mathcal{D}_{k,t}(i) di \quad (6)$$

for  $t = 0, 1, 2, \dots$ , where  $L_t$  denotes the aggregate labor bundle,  $W_t$  is nominal wage,  $B_t$  represents purchases of one-period discount bonds with interest  $I_t$  and  $\sum_{k=1}^K \mathcal{D}_{k,t} = \sum_{k=1}^K \int_0^1 \Pi_{k,t}(i) di$  are aggregate dividends received from the ownership of all firms in the economy.

The household must decide on how to allocate its' consumption expenditure among the different goods. The solution to this cost minimization of the aggregate consumption bundles yields the sectoral demand functions

$$C_{k,t} = \vartheta_k \frac{P_t}{P_{k,t}} C_t \quad (7)$$

where the aggregate price index is  $P_t = \prod_{k=1}^K \left( \frac{P_{k,t}}{\vartheta_k} \right)^{\vartheta_k}$ . Similarly, cost minimization of the sectoral consumption bundles yields demand for the good of firm  $i$  in sector  $k$

$$C_{k,t}(i) = \left( \frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\epsilon_k} C_{k,t} \quad (8)$$

where sectoral price indices are  $P_{k,t} = \left( \int_0^1 P_{k,t}(i)^{1-\epsilon_k} di \right)^{\frac{1}{1-\epsilon_k}}$  and where  $P_{k,t}(i)$  denotes the price of an individual firm  $i$  in sector  $k$ . Moreover,  $P_t C_t = \sum_{k=1}^K \int_0^1 P_{k,j,t} C_{k,j,t} dj$ . Eventually, optimal allocation of sectoral labor gives labor supply

$$L_{k,t} = \left( \frac{W_{k,t}}{W_t} \right)^\nu L_t \quad (9)$$

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<sup>8</sup>The idea behind this specification is “to capture some degree of specificity of labor while not deviating from the representative consumer/worker assumption (Horvath, 2000, p. 76)”.

where  $W_t = \left[ (1/K) \sum_{k=1}^K W_{k,t}^{1+v} \right]^{1/(1+v)}$ . Given the solution to the cost minimization problems, the problem of the household reduces to choose consumption  $C_t$ , labor  $L_t$  and savings  $B_t$  to maximize lifetime utility subject to the budget constraint (6). The solution is described by the optimality conditions concerning labor supply and intertemporal consumption choices

$$\frac{W_t}{P_t} = L_t^\varphi C_t^\sigma \quad (10)$$

$$1 = \mathbb{E}_t \left[ \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{R_t}{\Pi_{t+1}} \right] \quad (11)$$

where  $\Pi_{t+1}$  is the gross inflation rate of the aggregate price index between  $t$  and  $t + 1$ .

### 3.2 Firms

Firms assemble differentiated varieties of output using labor and intermediate inputs. The goods produced are then sold as an aggregate consumption bundle to households and as intermediate goods to other producers. In each sector,  $k = 1, \dots, K$ , there is a continuum of monopolistically competitive producers indexed by  $i \in [0, 1]$ . Within a sector  $k$ , each firm  $i$  produces with the same Cobb-Douglas production function that combines labor input and sector-specific intermediate inputs

$$Q_{k,t}(i) = L_{k,t}(i)^{1-\gamma_k} \left( \prod_{r=1}^K X_{k,r,t}(i)^{\omega_{k,r}} \right)^{\gamma_k} \quad (12)$$

where  $Q_{k,t}(i)$  is gross output by this producer,  $L_{k,t}(i)$  is labor used by firm  $(k, i)$ ,  $\gamma_k$  denotes the share of constant intermediate good use in the sector,  $\omega_{k,r}$  is the relative intensity with which firms in sector  $k$  use goods produced in sector  $r$  (Input-Output shares). I assume that  $\sum_{r=1}^K \omega_{k,r} = 1 \forall k$ . The  $K$ -by- $K$  matrix containing the shares of intermediate input use gives the representation of the production network, denoted by  $\mathbf{W}$ . The CES - aggregator of intermediate goods purchased by firm  $(k, i)$  from all firms in sector  $r$ ,  $X_{k,r,t}(i)$ , is given by

$$X_{k,r,t}(i) = \left( \int_0^1 X_{k,r,t}(i, j)^{\frac{\epsilon_k - 1}{\epsilon_k}} dj \right)^{\epsilon_k / \epsilon_k - 1}$$

where  $X_{k,r,t}(i, j)$  is intermediate inputs purchased by firm  $(k, i)$  from firm  $j$  in sector  $r$ .

The firm's problem can then be solved in two steps. First, finding the optimal mix of inputs for a given output price that minimizes costs and then finding the optimal price a firm would set given these inputs.

### 3.2.1 Marginal Costs

Each firm  $i$  in sector  $k$  faces the following cost minimization problem subject to expenditure minimization of sectoral inputs, where costs are given by

$$\mathcal{C}(Q_{k,t}(i)) = \min_{L_{k,t}(i), \{X_{k,r,t}(i)\}_r} W_{k,t} L_{k,t}(i) + \sum_{r=1}^K P_{r,t} X_{k,r,t}(i) \quad (13)$$

subject to the production function (12). Due to the CRS technology, the cost minimization problem for firm  $(k, i)$  can be rewritten in sectoral variables only and marginal costs are the same for all firms within the same sector. The price index for sectoral intermediate inputs is the same as for sectoral consumption goods by the assumption of same elasticity of substitution in consumption and production. It yields the following formula for the nominal marginal costs of production in sector  $k$

$$MC_{k,t} = \left( \frac{W_{k,t}}{1 - \gamma_k} \right)^{1 - \gamma_k} \left( \frac{P_t^k}{\gamma_k} \right)^{\gamma_k} \quad (14)$$

where  $P_t^k$  is the industry specific price index of intermediate inputs given by

$$P_t^k = \prod_{r=1}^K \left( \frac{P_{r,t}}{\omega_{k,r}} \right)^{\omega_{k,r}}. \quad (15)$$

The cost minimization has implications for firms' conditional factor demands. The firm's optimal choice of inputs, labor and gross output, given input prices, are

$$W_{k,t} L_{k,t}(i) = (1 - \gamma_k) \frac{MC_{k,t}}{P_{k,t}} P_{k,t} Q_{k,t}(i), \quad (16)$$

$$P_{r,t} X_{k,r,t}(i) = \gamma_k \omega_{k,r} \frac{MC_{k,t}}{P_{k,t}} P_{k,t} Q_{k,t}(i). \quad (17)$$

Expenditure on labor input or any particular intermediate input  $r$  is proportional to the firm's total expenditure.

### 3.2.2 Market Clearing

I can derive the total demand for goods produced by firm  $(i, k)$ . This firm can either sell its product as consumption goods to the representative household,  $C_{k,t}(i)$ , or as intermediate input to all firms from all sectors of the economy  $X_{r,k,t}(j, i)$ . This implies the following market clearing conditions

$$Q_{k,t}(i) = C_{k,t}(i) + \sum_{r=1}^K \int_0^1 X_{r,k,t}(j, i) dj \quad (18)$$

I can use the optimality conditions from the expenditure problems to replacing the CES aggregates  $C_{k,t}(i) = \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\epsilon_k} C_{k,t}$  and  $X_{r,k,t}(j, i) = \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\epsilon_k} X_{r,k,t}(j)$  to derive a demand for sectoral gross output

$$Q_{k,t}(i) = \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\epsilon_k} Q_{k,t} \quad (19)$$

where  $Q_{k,t}$  is sectoral gross output. It is defined as

$$Q_{k,t} = C_{k,t} + \sum_{r=1}^K X_{r,k,t} \quad (20)$$

where  $X_{r,k,t} = \int_0^1 X_{r,k,t}(j) dj$  is the total demand of sector  $r$  from inputs from sector  $k$ .

### 3.2.3 Price-Setting

Price-setting is modeled as in Calvo (1983), but with sector-specific probabilities as in Carvalho (2006). In particular,  $(1 - \theta_k)$  is the probability to reset prices in sector  $k$ . The problem of the firm is then to choose the optimal price  $P_{k,t}(i)$  to maximize the current market value of the profits generated while the price remains effective. Formally, firm  $(i, k)$  solves the problem

$$\max_{P_{k,t}(i)} E_t \left[ \sum_{s=0}^{\infty} (\theta_k)^s \Lambda_{t,t+s} Q_{k,t+s}(i) [P_{k,t+s}(i) - MC_{k,t+s|s}(i)] \right] \quad (21)$$

subject to firm demand (19) and where  $\Lambda_{t,t+s}$  is the stochastic discount factor implied by the household problem. The first-order condition is then given by

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \theta_k^s Q_{k,t+s}(i) \left[ P_{k,t}^* - \frac{\epsilon_k}{\epsilon_k - 1} MC_{k,t+s} \right]$$

where  $P_{k,t}^*$  is the optimal sectoral price, and  $\mu_k = \epsilon_k / (\epsilon_k - 1)$  is the sectoral markup absent nominal rigidities. Thus, firms resetting their prices will choose a price that equals the markup over their current and expected marginal costs, where the weights depend on the discount rate of the economy and the probability of the price of the firm remaining unset until each respective horizon.

Defining relative prices as  $p_{k,t}^* = \frac{P_{k,t}^*}{P_t}$  and rewriting the optimality condition in the standard recursive form, yields

$$p_{k,t}^* = \frac{\epsilon_k}{\epsilon_k - 1} \frac{\psi_{k,t}}{\Delta_{k,t}} \quad (22)$$

where  $\psi_{k,t}$  and  $\Delta_{k,t}$  are auxiliary variables that represent expected discounted values of marginal costs and revenues. They are defined recursively as

$$\psi_{k,t} = Q_{k,t} C_t^{-\sigma} m c_{k,t} + \theta_k \beta \mathbb{E}_t \left[ \Pi_{k,t+1}^{\epsilon_k} \psi_{k,t+1} \right] \quad (23)$$

$$\Delta_{k,t} = Q_{k,t} C_t^{-\sigma} + \theta_k \beta \mathbb{E}_t \left[ \frac{\Pi_{k,t+1}^{\epsilon_k}}{\Pi_{t+1}} \Delta_{k,t+1} \right] \quad (24)$$

where  $m c_{k,t} = M C_{k,t} / P_t$  are real marginal costs and  $\Pi_{k,t-1,t+s} = P_{k,t+s} / P_{k,t-1}$  is the gross nominal inflation rate.

The Calvo environment implies that sectoral price dynamics are described by

$$P_{k,t} = \left[ (1 - \theta_k) (P_{k,t}^*)^{1-\epsilon_k} + \theta_k (P_{k,t-1})^{1-\epsilon_k} \right]^{1/1-\epsilon_k}. \quad (25)$$

### 3.3 Monetary Policy

The monetary authority sets the short-term nominal interest rate,  $I_t$ , according to the following Taylor rule in accordance with value-added output and aggregate inflation

$$\frac{I_t}{\bar{I}} = \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_c} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} e^{v_t} \quad (26)$$

where  $\Pi_t = \prod_{k=1}^K \Pi_{k,t}^{\theta_k}$  is the aggregate inflation rate, variables with a bar denote steady state values,  $Y_t$  is aggregate nominal value-added and  $v_t$  is a monetary policy shock that follows an AR(1) process. In this model, the respective real measure is the value-added output,  $Y_t$  instead of gross output,  $Q_t$  and the inflation index that is relevant for consumption of households is,  $\Pi_t$ . The coefficients  $\phi_c$  and  $\phi_\pi$  measure the degree to which the monetary authority adjusts the nominal interest rate in response to changes in the consumption-based inflation rate and to changes in the value-added output respectively.

### 3.4 Equilibrium

Before turning to the log-linearized solution of the model and the Phillips curve in particular, in this section, I describe the model's equilibrium system. In particular, I stress its properties related to the determinants of marginal costs and the connection between gross output and value-added output. The equilibrium is described by the optimality conditions of the firms, the households along with market clearing conditions.

#### 3.4.1 Aggregation and value-added Output

In this economy, real aggregate value-added (i.e. real GDP),  $Y_t$  is equal to consumption,  $C_t$ . Let  $\mathcal{Y}_{k,t}(i)$  denote the nominal value-added of producer  $i$  in sector  $k$ . It is defined as the value of gross output produced by this firm abstracting the value of intermediate inputs it is using, i.e.

$$\mathcal{Y}_{k,t}(i) = P_{k,t}(i)Q_{k,t}(i) - \sum_{r=1}^K P_{r,t}X_{k,r,t}(i) \quad (27)$$

Aggregating over all real value-added of all producers in sector  $k$

$$\mathcal{Y}_{k,t} = \int_0^1 \mathcal{Y}_{k,t}(i)di = P_{k,t}Q_{k,t} - \sum_{r=1}^K P_{r,t}X_{k,r,t} \quad (28)$$

where  $X_{k,r,t} = \int_0^1 X_{k,r,t}(i)di$  by intermediate input clearing condition.

I can aggregate nominal dividends by using

$$\mathcal{D}_{k,t} = \int_0^1 D_{k,t}(i)di = P_{k,t}Q_{k,t} - W_{k,t}L_{k,t} - \sum_{r=1}^K P_{r,t}X_{k,r,t} = \mathcal{Y}_{k,t} - W_{k,t}L_{k,t}. \quad (29)$$

where I use the labor market clearing condition  $L_{k,t} = \int_0^1 L_{k,t}(i)di$ .

Eventually, substituting into the households' budget constraint (6), aggregate dividends and the bond market clearing  $B_t = 0$ , I obtain

$$P_t C_t = \sum_{k=1}^K W_{k,t}L_{k,t} + \sum_{k=1}^K \mathcal{D}_{k,t} = \sum_{k=1}^K \mathcal{Y}_{k,t}. \quad (30)$$

The aggregate nominal value-added equals the nominal value of total consumption. Real aggregate value-added (i.e. real GDP),  $Y_t$ , can then be derived by deflating the nominal aggregate value-added by the aggregate consumption price index,  $P_t$ , that is

$$Y_t = \frac{\sum_{k=1}^K \mathcal{Y}_{k,t}}{P_t} = C_t. \quad (31)$$

### 3.4.2 Wages and Total Expenditure

The role of total production in affecting marginal costs becomes clearer when deriving the optimal wages in this economy. Combining the labor market clearing condition  $L_{k,t} = \int_0^1 L_{k,t}(i)di$  with labor demand from firms (16) yields sectoral labor demand

$$L_{k,t} = (1 - \gamma_k) \frac{mC_{k,t}}{p_{k,t}} p_{k,t} Q_{k,t} d_{k,t} w_{k,t}^{-1} \quad (32)$$

where  $mC_{k,t} = \frac{MC_{k,t}}{P_t}$ ,  $p_{k,t} = \frac{P_{k,t}}{P_t}$ , and  $w_{k,t} = \frac{W_{k,t}}{P_t}$  are real sectoral marginal costs, relative sectoral prices and real sectoral wages respectively. Moreover,  $d_{k,t} = \int_0^1 \left( \frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\epsilon_k} di$  is within sector price dispersion. This corresponds to price dispersion in a one-sector model and as shown in Gali (2015), around a zero inflation steady state, price dispersion is approximately zero. Thus, for expositional purposes, I will not carry it along in the derivations that follow but have in

mind that it becomes negligible up to a first order approximation.<sup>9</sup> The exposition shows that sectoral labor demand depends on the total sectoral expenditure of the sector. This expenditure is the share of total revenue from production that is not spent on markups. Labor demand is increasing in the real value of sectoral production and decreasing in sectoral markups.

One can combine labor supply (10) and labor demand (16) with marginal costs (14) to solve for real wages

$$w_{k,t} = \frac{W_{k,t}}{P_t} = \left( \left( \frac{1-\gamma_k}{\gamma_k} \right)^{\gamma_k} (p_t^k)^{\gamma_k} Q_{k,t} \right)^{\frac{\varphi}{1+\gamma_k\varphi}} C_t^{\frac{\sigma}{1+\gamma_k\varphi}} \quad (33)$$

The wage rate depends on firms demand for labor input and households supply of labor via the following mechanisms. Wages increase in firms demand for labor input if their total production increases,  $Q_{k,t}$ , and in the cost of intermediate goods,  $p_t^k$ , as they can substitute labor inputs for intermediate inputs. On the other hand, wages increase in households labor supply if their demand for the aggregate consumption good,  $C_t$ , increases or the disutility from working,  $\varphi$ , falls.

### 3.4.3 Marginal costs

Replacing wages (33) in marginal costs (14), we can show that the average marginal cost in sector  $k$  yields

$$mc_{k,t} = \phi_k Y_t^{\frac{(\sigma+\varphi)(1-\gamma_k)}{1+\gamma_k\varphi}} \left( \frac{p_{k,t} Q_{k,t}}{Y_t} \right)^{\frac{\varphi(1-\gamma_k)}{1+\gamma_k\varphi}} \left( \frac{p_t^k}{p_{k,t}} \right)^{\frac{(1+\varphi)\gamma_k}{1+\gamma_k\varphi}} \quad (34)$$

where  $\phi_k = \frac{1}{1-\gamma_k} \left( \frac{1-\gamma_k}{\gamma_k} \right)^{\frac{\gamma_k(1+\varphi)}{1+\gamma_k\varphi}}$  is a constant.

In this economy, marginal costs are affected by three components: (i) the aggregate demand channel,  $Y_t$ , (ii) the real value of sectoral output and (iii) the price of intermediate inputs. While the first channel is standard, the other two are due to production networks. This is, however, only a partial equilibrium analysis since sectoral output depends on fluctuations in markups.

### 3.4.4 Linking Sectoral Production to GDP

The next step in my derivation is to solve for the real value of sectoral production,  $p_{k,t} Q_{k,t}$ , in terms of real value-added,  $Y_t$ . In particular, I will show that sectoral markups will affect other industries through the production network channel. I use the definition of real value-added, the market clearing condition (14) and the budget constraint (6) to obtain the following

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<sup>9</sup>It will become relevant under the assumptions of positive trend inflation (Ascari and Sbordone, 2014). Then this will introduce the propagation of sectoral price dispersions as discussed in Cienfuegos (2018).



characterization for the real value of sectoral production in terms of real aggregate value-added, i.e. GDP.

**Proposition 1.** *Let  $Q_t = \sum_r p_{r,t} Q_{r,t}$  denote the real value of total production (gross output),  $\delta_{k,t} = p_{k,t} Q_{k,t} / Q_t$  be the output share of sector  $k$  and  $\Phi_t^{NM} = Q_t / Y_t$  the network multiplier of the economy. The real value of sectoral production in the multi-sector economy with production networks and nominal frictions is linked to aggregate real value-added and given by*

$$p_{k,t} Q_{k,t} = \delta_{k,t} \Phi_t^{NM} Y_t \quad (35)$$

with

$$\Phi_t^{NM} \left( \frac{\overrightarrow{1}}{\mathcal{M}_t}, \delta_t \right) = \left[ 1 - \mathbf{1}' (\gamma \odot \frac{\overrightarrow{1}}{\mathcal{M}_t} \odot \delta_t) \right]^{-1} \quad (36)$$

$$\delta_t \left( \frac{\overrightarrow{1}}{\mathcal{M}_t} \right) = \left[ \mathcal{I} - (W' - V_C \mathbf{1}') \left( \mathbf{1} \left( \gamma \odot \frac{\overrightarrow{1}}{\mathcal{M}_t} \right)' \right) \right]^{-1} V_C \quad (37)$$

where  $\odot$  denotes the Hadamard (entrywise) product and  $\frac{\overrightarrow{1}}{\mathcal{M}_t}$  the  $K \times 1$  vector of sectoral real marginal cost deflated by sectoral prices which is also related to the inverse of sectoral markups  $\frac{1}{\mathcal{M}_{k,t}} = \frac{mc_{k,t}}{p_{k,t}}$ . Also,  $\gamma$ ,  $V_C$  and  $\delta_t$  are  $K \times 1$  vectors of sectoral intermediate good shares, sectoral consumption shares and sectoral output shares.  $W'$  is the inverse of the input-output matrix, reflecting how much intermediate goods each sector  $k$  provides to all other sectors.

Proposition 1 states that the real value of sectoral production is proportional to real value-added and depends on the topology of the production network. The network structure is captured by two statistics, (i) the network multiplier of the economy and (ii) the output share of a sector. Those two statistics - and hence the relationship between sectoral production and value-added - are affected by variations of markups across sectors.

In the next two sections, I will provide intuition about the two network statistics and explain why we can think about the network multiplier as a measure of the importance of the network to the economy and the output share as a measure of centrality of a sector.

### 3.4.5 Network multiplier

The network multiplier,  $\Phi_t^{NM}$ , provides a link between real value-added production,  $Y_t$ , and the real value of total production (gross output)  $Q_t \equiv \sum_r p_{r,t} Q_{r,t}$ , in the economy. In a model without intermediate goods the multiplier would be one. In the multi-sector model with production networks, to produce one more unit of the aggregate consumption good, additional units

of production are produced that will be used as intermediate goods. Therefore, the network multiplier will be larger than one.

The larger the network multiplier, the more labor is needed in order to produce the same consumption unit. One can think about this as a proxy for the length of the production chain in this economy. The longer the production chain, the more intermediate goods and hence labor input is needed for production.

To derive the network multiplier, I rewrite the budget constraint (6) in terms of real value-added and the total value of production

$$Q_t = Y_t + \sum_{k=1}^K \gamma_k \frac{mc_{k,t}}{p_{k,t}} \delta_{k,t} Q_t. \quad (38)$$

where I use the definition of the output shares  $p_{k,t} Q_{k,t} = \delta_{k,t} Q_t$ . The real value of total production exceeds real aggregate value-added by the aggregate expenditure on intermediate goods.

This recursive expression of real total value of production can be iterated to obtain a representation capturing all direct and indirect expenditure effects along the production network

$$Q_t = Y_t + \underbrace{\sum_{k=1}^K \gamma_k \frac{mc_{k,t}}{p_{k,t}} \delta_{k,t} Y_t + \sum_{k=1}^K \gamma_k \frac{mc_{k,t}}{p_{k,t}} \delta_{k,t} \sum_{r=1}^K \gamma_r \frac{mc_{k,t}}{p_{k,t}} \delta_{r,t} Y_t + \dots}_{\text{network component}} \quad (39)$$

The first term captures household's expenditure on the consumption good. The second term captures the indirect expenditure on intermediate goods as a proportion of total consumption. The third and higher terms, reflect the expenditure on intermediate goods higher in the production chain. All of those indirect effects decay at the rate given by the intermediate good share,  $\gamma_k$ , and the markup,  $\frac{1}{\mathcal{M}_{(k,t)}}$ .<sup>10</sup> The size of the decay is weighted by the importance of a particular sector, reflected by its output share,  $\delta_{k,t}$ . The network component captures these indirect effects. It increases if large (in terms of output shares) sectors have larger intermediate good shares and smaller markups. Importantly, here we can see how distortions in markups can propagate through the network and change total expenditure. In the New Keynesian model, markups are countercyclical, which means that the network component and hence the network multiplier increases in booms.

The decay of the network multiplier has another intriguing interpretation. The smaller the decay, the longer will be the production chain. If the network structure is such that more central

<sup>10</sup>If we assume the intermediate share to be constant,  $\gamma_k = \gamma$ , we would find

$$Q_t = Y_t + \gamma \sum_{k=1}^K \frac{mc_{k,t}}{p_{k,t}} \delta_{k,t} Y_t + \gamma^2 \sum_{k=1}^K \frac{mc_{k,t}}{p_{k,t}} \delta_{k,t} \sum_{r=1}^K \frac{mc_{k,t}}{p_{k,t}} \delta_{r,t} Y_t + \dots \quad (40)$$

i.e. higher order terms are devaluated by the rate  $\gamma$ .

sectors have larger intermediate shares, then the total decay of the network will be smaller, and the total multiplier larger. I will investigate this mechanism more when I look at different examples of networks.

Rewriting the sums in vector form, I can rewrite the last expression as

$$Q_t = \underbrace{\left[ 1 - \mathbf{1}' \left( \gamma \odot \frac{\overrightarrow{1}}{\mathcal{M}_t} \odot \delta_t \right) \right]^{-1}}_{=\Phi_t^{NM}} Y_t \quad (41)$$

where  $\gamma$ ,  $\frac{\overrightarrow{1}}{\mathcal{M}_t}$  and  $\delta_t$  are  $K \times 1$  vectors of sectoral intermediate good shares, sectoral markups and sectoral output shares, and  $Q_t$  as well as  $Y_t$  are scalars.

The network multiplier,  $\Phi_t^{NM}$ , shows how much total production is needed in order for the household to consume one unit of the consumption good. By capturing the size of the network component relative to the direct expenditure on the consumption good, it is therefore a measure of the relative importance of the network in this economy. As long as intermediate good shares are positive, the network component will be non-zero and the network multiplier will be larger than one. The network multiplier is not constant as it depends on markups and the output shares. In the next section, I develop a closed-form expression for output shares.

### 3.4.6 Output Shares

The output share,  $\delta_{k,t}$ , provides a link between the real value of sectoral production and the real value of total production. In a model without intermediate goods, the output share will be equal to the consumption share,  $V_C$ . In the multi-sector model with production networks, however, each sector also provides intermediate goods to other sectors. From the system of market clearing conditions (20) in combination with sectoral intermediate good demands (17), the demand for sectoral production yields

$$p_{k,t} Q_{k,t} = \vartheta_k C_t + \sum_{r=1}^K \omega_{r,k} \gamma_r \frac{m_{C_{r,t}}}{p_{r,t}} p_{r,t} Q_{r,t} \quad (42)$$

where the first part on the right hand side represents the household's direct demand for goods from sector  $k$ . This demand is fully described by the consumption share of goods from sector  $k$  in the total demand of the household for the aggregate consumption good,  $\vartheta_k$ , i.e. it reflects the preferences of the household.<sup>11</sup> The second component is the demand from other sectors  $r$  that use sector  $k$ 's good as an intermediate input. It is given by the share sector  $r$  spends on goods from sector  $k$ ,  $\omega_{r,k}$ , relative to its total intermediate good expenditure.

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<sup>11</sup>By the Cobb-Douglas assumption on the consumption aggregator, this is thus a constant fraction of total consumption.

Combining with the budget constraint (6) and dividing by the total real value of production yields an iterated representation of the output shares

$$\delta_{k,t} = \vartheta_k + \underbrace{\sum_{r=1}^K \tilde{\omega}_{r,k} \vartheta_r + \sum_{r=1}^K \sum_{s=1}^K \tilde{\omega}_{s,r} \tilde{\omega}_{r,k} \vartheta_s + \dots}_{\text{network component}} \quad (43)$$

where  $\tilde{\omega}_{r,k,t} = (\omega_{r,k} - \vartheta_k) \gamma_r \frac{1}{\mathcal{M}_{r,t}}$  is a weighting matrix.

The first term represents again the direct demand for goods from sector  $k$  from the household. The first term of the network component captures the importance of sector  $k$  to its immediate customers, firms that are directly connected to  $k$ . The specific contribution of sector  $r$  to  $k$ 's output share depends on sector  $r$ 's own share,  $\delta_{r,t}$ , and on a weighting matrix,  $\tilde{\omega}_{r,k}$ . This weighting matrix depends on the network weight connecting both sectors,  $\omega_{r,k}$  and the intermediate good expenditure of sector  $r$ . The second term of the network component captures the indirect importance of  $k$  through sectors that buy inputs from sector  $k$ 's customers. In other words, this is the indirect demand from the customers of the customers of sector  $k$ . The third and higher order terms capture the importance of  $k$  through customers that are one or more further steps away from sector  $k$ .

Again, we can rewrite the last equation in vector form to represent the relationship between the vector of sectoral real values of production,  $\vec{PQ}_t$ , and the total real value of production,  $Q_t$ , in this economy

$$\vec{PQ}_t = \underbrace{(\mathcal{I} - \tilde{W}_t)^{-1} V_C}_{=\delta_t} Q_t \quad (44)$$

where  $\tilde{W}_t = (W' - V_C \mathbf{1}') (\mathbf{1} \odot \frac{\vec{1}}{\mathcal{M}_t})'$ .

The output share summarizes the network structure by specifying how much sectoral production  $p_{k,t} Q_{k,t}$  is needed in order to satisfy a given demand for gross output,  $Q_t = \sum_r p_{r,t} Q_{r,t}$ . In fact, output shares are equal to the *network centrality of Katz* (1953), i.e. they capture the relative importance of each node (sector) in a network (aggregate economy). A sector is important if its outdegree is larger than its consumption share, i.e.  $(W'_k - V_{C,k} > 0)$ , and if its customers have large intermediate good shares,  $\gamma_k$ , or small markups,  $\mathcal{M}_{k,t}$ . In the absence of intermediate goods or markups, the output share is equal to the consumption share,  $V_C$ . Again, fluctuations in markups in other sectors are transmitted through the network via intermediate good use,  $W$ .

### 3.5 Steady State and Network Examples

In this section, I highlight some key features of network statistics in steady state. Therefore, I examine some examples of network structures in order to provide intuition about the network

statistics introduced before. Moreover, I discuss how both statistics can be directly calculated from the data.

The main difference between a zero-inflation steady state and the equilibrium is that markups are constant and given by  $\bar{\mathcal{M}} = \frac{\epsilon_k}{\epsilon_k - 1}$ . Evaluating network statistics and marginal costs in steady state yields

$$\bar{\Phi}^{NM} = \left[ \mathbf{1} - \mathbf{1}' \left( \gamma \odot \frac{\vec{\mathbf{1}}}{\bar{\mathcal{M}}} \odot \delta \right) \right]^{-1} \quad (45)$$

$$\bar{\delta} = \left[ \mathcal{I} - (W' - V_C \mathbf{1}') \left( \mathbf{1} \left( \gamma \odot \frac{\vec{\mathbf{1}}}{\bar{\mathcal{M}}} \right)' \right) \right]^{-1} V_C \quad (46)$$

This expression shows that the two networks statistics are depend on network characteristics only and the sectoral markups. Independent of nominal rigidities. In the dynamic model (i.e. outside the steady state), the markups will vary and thus the network multiplier.

Another interesting implication comes from these equations. Given observables for intermediate good shares, value-added shares, the input-output tables and markups, we can calculate both network characteristics directly (without additional assumptions) from the data. Actually, the output shares even shave a direct empirical counterpart in the data. They can be observed at the yearly frequency from input output tables. Thus, they could potentially be used as sufficient statistics for the slope of the PC.

Before I turn to the calibration part, where I show how the network statistics have changed over time, I will outline how different features of hypothetical network economies affect the two network statistics.

**EXAMPLE 1 (Change in network multiplier):** This economy has two sectors. I assume that the household preference weights on each good are the same,  $V_C = [0.5; 0.5]$ . Markups are symmetric and equal to 20%, i.e.  $\frac{1}{\bar{\mathcal{M}}} = 5/6$ . Moreover, I consider two networks that differ in their intermediate shares

$$W_1 = W_2 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \text{ and } \gamma_1 = \begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix} \text{ and } \gamma_2 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

Both networks feature a symmetric roundabout production network: Sectors 1 and 2 equally spend their input expenditure on inputs from sectors 1 and 2. The equivalence of consumption shares and outdegrees implies that both sectors have the same output share in steady state ( $\delta = V_C$ ), with each sector having 50% of the market. However, Network 2 has a smaller network multiplier of 1.71 because of its lower intermediate share than Network 1 of 2.67.

In this example, the network multiplier changes without changes in output shares. This will

become important later to decompose changes in the slope through the lens of a symmetric network model.

**EXAMPLE 2 (A (non-)“irrelevant” sector):** Keeping the markup structure of Example 1, and adjusting the household preferences such that sector 1 becomes irrelevant for households  $V_C = [0; 1]$ , Network 3 and is given by

$$W_3 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \text{ and } \gamma_3 = \begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix}$$

This networks is a representation of a star network: Sectors 1 and 2 spend all of their input expenditure on inputs from sectors 1. Therefore, sector 1 that is irrelevant from the household perspective, is the central sector of this economy from the network perspective: Sector 1 has non-zero output shares as  $\delta = [0.63; 0.37] \neq V_C$ . By the symmetry of the intermediate good shares and markups, Network 3 has the same network multiplier as Network 1, 2.67.

Hence, consumption weights do no longer characterize the importance of a sector for the economy. This is basically an illustration that Hulten’s law (1978) does not hold in this economy. The impact of a sectoral TFP shock is not equal to the sector’s share in total value-added.

**EXAMPLE 3 (Change in output share):** In the next example, I want to show how the change of the I/O structure can change the output shares. Keeping the consumption shares, and intermediate good shares from Network 3 but increasing the markup from Sector 1 to 50%, i.e.  $\frac{1}{\mathcal{M}} = [3/6; 5/6]$ , I will look at the transition from Network 4 to Network 3, where the former is given by

$$W_4 = \begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \end{pmatrix}$$

In Network 4, Sector 2 is equally spending its expenditure on inputs from both sectors instead of solely using inputs from Sector 1 (Network 3). Therefore, Sector 1 will be relevant again but with a lower output share  $\delta_4 = [1/3; 2/3]$ . Transitioning to Network 3, the outdegree of Sector 1 increases and so does its output share to  $\delta_3 = [1/2; 1/2]$ . The increase in the centrality has another interesting implication in this example. Since, Sector 1 has a higher markup, the aggregate output share weighted markup increases. This decreases the network multiplier from 2.18 to 2.<sup>12</sup>

**EXAMPLE 4 (Increasing the “length” of the production chain):** In the last example, I want to show how the change of the I/O structure can increase the network multiplier by increasing the length of the network. Keeping the consumption shares, and markups from Network

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<sup>12</sup>This is also smaller as the multiplier in the Network 3 in Example 2, of 2.67, because in Example 2 both sectors had lower markups.

1, I will consider the following two networks

$$W_5 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \text{ vis-a-vis } W_6 = \begin{pmatrix} 0.9 & 0.1 \\ 0.9 & 0.1 \end{pmatrix} \text{ and } \gamma_5 = \gamma_6 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

While Network 5 is a symmetric network again as in example 1, Network 6 is a star network with Sector 1 being the central sector. Increasing the centrality of the sector that uses more intermediate goods can be seen as increasing the overall length of that network as discussed in Section 3.4.6. Making Sector 1 the “star” in the star network increases its centrality from 0.5 to 0.73 as measured by its output share. This increases the network multiplier from 2.08 to 2.32.

### 3.6 Equilibrium conditions and Dynamics

The equilibrium is described by a system of  $7N + 3$  equations to pin down the  $7N + 3$  endogenous variables: sectoral variables  $\{\Pi_{k,t}, p_{k,t}, Q_{k,t}, mc_{k,t}, d_{k,t}, \psi_{k,t}, \Delta_{k,t}\}_{k=1}^K$  and aggregate  $\{Y_t, I_t, \Pi_t\}$  variables given the monetary shocks  $e^{v_t}$ . The three aggregate equations are the Euler equation (11), the Taylor rule (26) and the aggregate labor supply (10). The sectoral equations are output demand (20), labor demand and marginal costs (14) those that determine the optimal pricing decision (22), (23) and (24).

## 4 Calibration and Structural Transformation

This section describes the baseline calibration of the model and the data sources. One of the objectives of the calibration is to compute the model’s implied slope of the New Keynesian Phillips curve over time. I will allow for time-variation in the calibration of different parameters: the production network, i.e. (i) the composition of sectoral intermediate goods,  $W$ , which will be derived from the Input-Output tables, as well as (ii) sectoral intermediate good shares,  $\gamma_k$ , and (iii) the size of each sector as measured by its’ value-added share,  $\vartheta_k$ .

In the second part, I outline how the production network in the U.S. has structurally changed in the past decades as represented by the two network statistics introduced in the last section: (i) network multiplier and (ii) output shares. In this respect, I show that services have not only become more central in value-added terms but also with respect to the network structure. It follows a discussion of examples and characteristics that describe those sectors that have become most central.

### 4.1 Calibration

Starting from the sectoral definitions of the “summary level” of the Input-Output tables from the BEA, I excluded the government sector to be consistent with the model. Moreover, the specification of sectors has changed in from 1996 to 1997. To account for these changes in the

classification, I merged 5 sectors to be consistent with the 1963 specification. Eventually, the dataset covers 53 sectors at roughly the 3-digit NAICS level from 1963 to 2017.

#### 4.1.1 Production network

I use data from the Bureau of Economic Analysis (BEA) on the flow of goods from each industry in the U.S. economy to other industries. The aggregated industries defined by the BEA sum to gross domestic product and therefore cover the entire economy. The Input-output tables are available at an annual frequency and show the dollar value of goods produced, for example, in industry  $i$  that industry  $j$  uses as inputs. For each sector, I use this information to derive the composition of intermediate goods  $\omega_{k,r}$ , final demand  $\vartheta_{,k}$  as well as the intermediate goods share,  $\gamma_k$ .

#### 4.1.2 Frequency of Price Changes

The frequency of price changes is calibrated using data from Pasten, Schoenle and Weber (2016). They calculate monthly frequencies using confidential microdata underlying the BLSs Producer Price Index (PPI). Based on these frequencies of price adjustments at the goods level, they aggregate these into the 350-sector industry level industry definitions of the Bureau of Economic Analysis (BEA). In order to map them into the 53 sector specification, I compute the median frequency within each 3-digit sector. The monthly frequencies are transformed to match the quarterly calibration of the model.

#### 4.1.3 Other Parameters

I calibrate the model at the quarterly frequency using standard parameter values in the literature. The discount factor is assumed to be  $\beta = 0.99$  which implies a annual steady state return on financial assets of about 4 percent. It is also assumed a unitary intertemporal elasticity of substitution and inverse Frisch elasticity of labor supply as well as labor mobility  $\sigma = \nu = \varphi = 1$ . As to the interest rate rule coefficients, it is assumed  $\phi_\pi = 1.5$  and  $\phi_c = 0.5/4$ . The persistence parameter of monetary shocks is  $\rho_m = 0.5$ . Finally the constant elasticity of substitution is set equal to  $\epsilon = 6$  in order to match a steady state markup of 20%.<sup>13</sup> Table 7 summarizes the calibration of the other business cycle parameters.

## 4.2 Structural Change in the Production Network

This section documents how the input-output network structure of the U.S. economy has changed over time. I use the input-output data from the BEA to analyze the U.S. economy over

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<sup>13</sup>In the empirical part of the paper, I abstract from heterogeneity in the elasticity of substitution in the benchmark case. However, an online appendix discusses an extension with heterogeneity.



a long time span (from 1963 to 2017) at an annual frequency. Importantly, I find that a sectoral change from manufacturing to services did not only take place for final demand, but also in terms of the network structure. Consistent with a process of service deepening (Galesi and Rachedi, 2016), I find that certain, usually services-based industries have become more important in terms of centrality or intermediate good provision in the network over time. In the second part of the section, I study how these changes are reflected in the two network statistics from Section 3.

#### 4.2.1 Changes in the Input-Output Structure over Time

First, I will look at changes to the network structure of the economy by comparing the input-output tables from 1963 to 2017 and then compare measures of network centrality. The Figure 5 displays the Input-output matrix as created by the guidelines of BEA handbook in 1963 and 2017, respectively. They shows the contribution from all the other sectors to the intermediate goods of sector  $k$  (vertical axis). There are three things to notice. First, sectors usually tend to consume a lot of intermediate goods from their own sector as represented by the dark shade of the diagonal line. Second, service sectors and manufacturing sectors prefer sectors of the same category. The figure displays accumulations of shaded boxes in the top left and bottom right quadrant of the heatmap. Over time, the pattern becomes weaker for the Manufacturing sector and the opposite for services. Third, from 1963 to 2017, there is a strong increase in the use of services across the board. This trend is particularly apparent when looking at Figure 5. However, two sectors stand as illustrated by a vertical line: (i) Management of companies and enterprises and Miscellaneous professional, scientific, and technical services (Man prof scie tech) and (55) and Administrative and support services (Administration).

#### 4.2.2 Which Industries have become more or less central?

In this section I analyse how the input-output relationships have changed over time by means of our first network characteristic: *output shares*. The output shares,  $\delta_k$ , are a measure of centrality, as outlined in Section 3. Centrality is one way of measuring the relative importance of each node (industry) in a graph (network). It is particularly important because centrality takes into account not only an industry’s connections to other industries but also the strength of these connections and how connected the other industries are. In this way, an industry will tend to have a high measure of centrality if it is connected to other industries with high centrality.

Table 7 shows the most and least central industries in terms of output shares at the beginning and end of the 1963-2017 sample. Of the top central firms in 1963 were three manufacturing industries with “Food, beverage and tobacco products”, “Motor vehicles, and parts” and “Chemical products”. Of those, only the first stayed in the Top 10. At the same time, among the least

central industries in 2017, we can find 4 Manufacturing firms, while in 1963, none was Manufacturing. In 2017, half of the most central industries were Services. Important contributors to this change is the increasing importance of the Health sector with “Hospitals and nursing and residential care facilities” or “Ambulatory Health care services” as well as the FIRE sector represented by “Real Estate” and “Insurance carriers and related activities”.

### 4.2.3 Which are the Characteristics of Sectors that increased a lot?

According to Table 7 the industries with the biggest rank declines were food and manufacturing related, while those with the biggest rank increases were almost exclusively services based. Looking at the characteristics of those sectors, we can observe that sectors with rank increases tend to have higher price rigidities (higher probability of not changing prices) and higher increases in their outdegree.<sup>14</sup> While they have relatively lower intermediate good shares, they display larger increases in those, too. Of those industries, “Management and Professional, scientific and technical services” is a good example. The industry is on the lower end of the frequency distribution (see Figure 9), and saw its’ outdegree double from 3.46 to 6.17.

Conversely, industries with rank decreases are almost exclusively manufacturing related (except for “Construction”, “Farms” and “Retail trade”), and are characterized by higher intermediate good shares and decreasing outdegrees. “Food, beverage and tobacco products” have an intermediate goods share of 0.71 and their outdegree fall by half from 1.63 to 0.85 in between 1963 and 2017. Among industries that became less central are industries with very small price rigidities such as “Farms” 0.02 or “Primary metals” with 0.2.

### 4.2.4 Does this resemble the Structural Transformation in Value-Added Shares?

Industries with high centrality do not necessarily also have high value-added. Looking at “Management, professional, scientific, and technical services” we see it is among the top rank increases in terms of centrality, however, the industry’s GDP share has stayed almost unchanged. The same can be observed for sectors that became less central. While “Primary metals”, “Machinery” and “Fabricated metal products” are among the industries that became less centrality, their GDP share did not change. Centrality measures an industry’s importance as part of the input-output network, not necessarily its importance within GDP. GDP counts only the amount of goods and services that go into into final uses, e.g. consumption or investment (value-added). GDP excludes input-output flows since intermediate goods are excluded from value-added output. Therefore, industries that are large providers of intermediates goods but not final goods, can have high centrality but low value-added, and vice versa. Another good example is “Oil and

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<sup>14</sup>The outdegree is an alternative measure of centrality and is central for the network statistics. The outdegree of sector  $j$  is defined as  $W'_j = \sum_{i=1}^K \omega_{i,j}$ .

gas extraction”. The industry does basically not provide goods for final demand, a value-added share of zero, but has an output share of 0.08% by being a large provider of intermediate goods.

Therefore, industries that are important from a network perspective may not necessarily be important from a GDP perspective. Conversely, just because an industry is important in terms of value-added does not imply that it has a large role in the input-output network.

### 4.3 Changes in the Distribution over Time

Finally, in this section, I will display how the whole distribution of industries has changed in terms of value-added shares, output shares and intermediate good shares. In general, we can observe again structural changes in the network structure (process of service deepening) additionally to structural transformation.

The Figure 10 shows the change in sectoral output shares as defined above. It is apparent how services became the most central sector in the economy. To put those numbers in context, the average centrality in 1963 was 1.9%. “Miscellaneous professional, scientific, and technical services” saw a gain in output share by 2.5 percentage points, i.e. increased by more than the average total size of a sector. The reason for this development is threefold. First, the outdegree of services increased. This means services provide more intermediate inputs to other industries. We can see this development from figure

Therefore, these results highlight the structural changes in the network structure from 1963 to 2017. There was not only a reallocation between sectors in terms of value-added shares (usually referred to as *structural transformation*), but also in terms of the centrality of sectors (*structural change in production networks*). In particular, Services became the most central sector in the economy (service deepening) and this had important implications on the sensitivity of inflation to the output gap, as discussed in the next section.

## 5 Inflation Dynamics

In this section I discuss how the production network is reflected in the Phillips curve. I will show how the multi-sector model with production networks compares to other models with respect to the sensitivity of inflation to the output gap. Before analyzing the impact of the structural changes in the network statistics identified in the previous section, I will derive the sensitivity in different cases in order to study the contribution of each feature of the model on inflation dynamics.

In order to derive the Phillips curve in this environment, I start by log-linearizing the models equilibrium conditions around a zero inflation steady state and analyze the resulting system of difference equations. Unless otherwise noted I use the  $\hat{\cdot}$  symbol on top of a variable to indicate the deviation from its steady state value.

## 5.1 The Sectoral Phillips Curves

As standard in New Keynesian models, inflation dynamics are described by a forward-looking relationship between inflation and markups. From the expression of markups from equation (34), these sectoral New Keynesian Phillips Curves (NKPC) from the model described in Section 3 are given by

$$\hat{\pi}_{k,t} = \beta \mathbb{E} \hat{\pi}_{k,t+1} + \kappa_k \frac{(1 - \gamma_k)}{1 + \gamma_k \varphi} (\sigma + \varphi) \hat{y}_t + \kappa_k \frac{\varphi(1 - \gamma_k)}{1 + \gamma_k \varphi} (\hat{p}_{k,t} + \hat{Q}_{k,t} - \hat{y}_t) - \kappa_k \frac{(1 + \varphi)}{1 + \gamma_k \varphi} (\gamma_k \hat{p}_t^k - \hat{p}_{k,t}) \quad (47)$$

where  $\kappa_k = (1 - \theta_k)(1 - \beta\theta_k)/\theta_k$ .

The expression shows that sectoral inflation dynamics in this economy depend on three variables additional to future expected inflation: (i) the output gap as in the standard NKPC, (ii) sectoral real value of sectoral production gaps,  $(\hat{p}_{k,t} + \hat{Q}_{k,t})$  and (iii) sectoral relative price gaps,  $(\hat{p}_{k,t} - \gamma_k \hat{p}_t^k)$ . The latter two variables are two channels that determine inflation but are ignored in simpler models.

The production network enters in three ways. First, the coefficients in front of the three variables depend on the sectoral intermediate share,  $\gamma_k$ . With more intensive intermediate good use, the first two channels (via the wage channel) become less important and marginal costs depend more on input price gaps,  $\hat{p}_t^k$ . Second, these relative input price gaps depend on the input-output network via  $W$  since they are defined by  $\hat{p}_t^k = \sum_r \omega_{k,r} \hat{p}_{r,t}$ . Three, as delineated in Section 3, sectoral production gaps,  $(\hat{p}_{k,t} + \hat{Q}_{k,t})$ , depend on the network structure via two statistics: (i) the network multiplier and (ii) output shares.

There are numerous implications from the sectoral Phillips curves in (47). First, the presence of relative price gaps in the sectoral NKPCs adds persistence into the inflation dynamics. In particular, relative prices are lagged endogenous variables, and, thus, they introduce a backward-looking component in the determination of inflation. This occurs for two reasons. Due to the multi-sector structure with heterogeneity in nominal frictions as in Woodford (2003), which is represented by  $\hat{p}_{k,t}$ . Additionally, the presence of intermediate goods in production, introduces relative price gaps via the input price gaps. Second, due the presence of relative price gaps and sectoral production in the determination of aggregate inflation, there will not be a so-called ‘‘divine coincidence’’ (Blanchard and Galí, 2007). Instead, the central bank will face a trade-off between the stabilization of inflation and output gap.

## 5.2 Aggregate Inflation Dynamics and the Slope of the Phillips Curve

Using the definition of the aggregate price index, the aggregate NKPC is a weighted average of the sectoral NKPC’s  $\hat{\pi}_t = \sum_k \omega_{ck} \hat{\pi}_{k,t}$  and given by

$$\begin{aligned}
\hat{\pi}_t = & \beta \mathbb{E} \hat{\pi}_{t+1} + \underbrace{\sum_k \omega_{ck} \kappa_k \frac{(1 - \gamma_k)}{1 + \gamma_k \varphi} (\sigma + \varphi) \hat{y}_t}_{\equiv \Phi^{Std}} + \underbrace{\sum_k \omega_{ck} \kappa_k \frac{\varphi(1 - \gamma_k)}{1 + \gamma_k \varphi} (\hat{p}_{k,t} + \hat{Q}_{k,t} - \hat{y}_t)}_{\equiv \Phi^{Multi}} \\
& + \underbrace{\sum_k \omega_{ck} \kappa_k \frac{(1 + \varphi)}{1 + \gamma_k \varphi} (\gamma_k \hat{p}_t^k) - \hat{p}_{k,t}}_{\equiv \Phi^{StratComp}}
\end{aligned} \tag{48}$$

According to this expression, aggregate inflation in this economy is determined by the sum of sectoral dynamics of the output gap,  $\Phi^{Std} \hat{y}_t$ , the weighted average of sectoral over production gaps (total value of sectoral production relative to value-added),  $\Phi^{Multi}$ , and aggregated relative price gaps,  $\Phi^{StratComp}$ . Additionally, as is standard in New Keynesian Phillips curves, inflation expectations matter,  $\beta \mathbb{E} \hat{\pi}_{t+1}$ .

What is the slope of the Phillips curve, i.e. the sensitivity of aggregate inflation to the output gap? Naturally, one would say the coefficient in front of the output gap. In the production network model, however, this answer is incomplete, because both relative price gaps and sectoral production gaps are correlated with the output gap. The slope of the Phillips curve will thus depend on the sensitivity of all three terms  $\Phi^{Std}$ ,  $\Phi^{Multi}$  and  $\Phi^{StratComp}$  with respect to the output gap. In each case, this is a combination of the three coefficients together with the correlation of the respective endogenous variable and the output gap. If this correlation is negative (coefficient is positive), then the term has a dampening effect on the slope. In order to find the slope of the Phillips curve, the correlations between the output gap and relative price gaps as well as sectoral production gaps need to be calculated.

What would an econometrician, who would want to estimate the sensitivity of inflation to the output gap, estimate? Estimating the model in Equation (1), he would estimate the sensitivity of inflation to the output gap as the combined effect of channels 1)-3). Therefore, any change in one of those channels can account for changes in the slope of the PC. If the econometrician would want to estimate  $\Phi^{Std}$  he would get a biased estimate by running the estimation in (1). Therefore, changes in the estimated slope over time (Section 2) can originate in any of the three components and not just in  $\Phi^{Std}$ .

A few additional things are noteworthy about the last term. It shows the presence of strategic complementarities in price-setting. When the optimal price chosen by a firm depends positively (negatively) on the prices of other firms, we speak of strategic complementarities (substitutes) (see Cooper and John, 1988)<sup>15</sup>. There are different sources for strategic complementarities such as kinked demand curves (Kimball, 1995) or factor attachments (see Basu, 2005). In the present

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<sup>15</sup>In the simple setting of CJ, strategic complementarity arises if the profit of a firm  $V(p_i, p_{-i})$  is such that  $V_{12}(p_i, p_{-i}) > 0$ .

case, strategic complementarities arise because of sticky intermediate good prices. In the production network setting, however, there are two distinct differences to strategic complementarities from general standard formulations of intermediate goods (Basu, 1995).

First, prices depend positively on the sector specific input price,  $p_t^k$  instead of the aggregate price level,  $p_t$ . As an implication, the degree of strategic complementarity will be sector specific and depend on the share of intermediate goods used in production. Second, due to the network structure also indirect effects will be present. The price-setting of a firm will depend on the prices of their suppliers. Since those, however, themselves use intermediate goods in production, their prices depend on the suppliers' suppliers' prices and so on. Consequently, the degree of strategic complementarity depends on the particular network structure of the whole economy. In particular, the interaction of suppliers' stickiness,  $\theta_k$ , and intermediate good share  $\gamma_k$  will impact downstream sectors; e.g. if an important supplier is rigid then its' downstream sectors will exhibit large degrees of strategic complementarity, too. This feature reduces the sensitivity of real marginal cost and hence aggregate inflation to changes in aggregate demand (real rigidity, Ball and Romer, 1990).

Before deriving the slopes in different models, in the next section, I will look at one special case of the economy from Section 3 and illustrate the implications for the Phillips curve and its slope.

**Multi-Sector Model without intermediate goods.** Here, I will derive the Phillips curve that arrives out of a multi-sector model without intermediate goods,  $\gamma_k = 0$ , but with heterogeneity in nominal rigidities (Woodford Ch. 3, 2003 or Carvalho, 2006). The aggregate inflation dynamics in this model are given by

$$\hat{\pi}_t = \mathbb{E} \hat{\pi}_{t+1} + \bar{\kappa}(\sigma + \varphi)\hat{y}_t - (1 + \varphi) \sum_k \omega_{ck} \kappa_k \hat{p}_{k,t} \quad (49)$$

where  $\bar{\kappa} = \sum_k \omega_{ck} \kappa_k$  is the average slope coefficient.

The second term on the right-hand side of Equation (49) corresponds to the slope coefficient with respect to the output gap,  $\Phi^{Std}$ . In the absence of intermediate goods, excess production is zero, i.e. the sectoral real value of production is equivalent to sectoral consumption. It is apparent from this expression that the econometrician will not recover  $\bar{\kappa}(\sigma + \varphi)$  as the slope of the Phillips curve but instead a biased estimate due to relative prices. The last term in Equation (49) is due to heterogeneity in nominal rigidities.<sup>16</sup> This induces an increase in monetary non-neutrality as in Carvalho (2006) or Carvalho and Schwartzmann (2015) because relative price gaps and the output gap are correlated. Only if the econometrician controls for changes in relative price gaps, will he recover the standard slope,  $\bar{\kappa}(\sigma + \varphi)$ .

<sup>16</sup>In the absence of heterogeneity in nominal rigidities, prices are symmetric:  $\sum_k \omega_{ck} \hat{p}_{k,t} = 0$ .

### 5.3 Implications for the Slope in Different Economies

In this section I calibrate the economy to the input-output structure of 2007. I will compute the implied slopes and compare the model's predictions for the three components of the Phillips curve. Moreover, I will provide a decomposition of the different components of the aggregate sensitivity of inflation to the output gap introduced in the last section. In a second step, I will estimate the sensitivity an econometrician would recover from simulated data in order to show this coincides with the sum of the three components. Finally, I compare the predictions of the model concerning the network statistics and dynamic multiplier.

**Different economies.** I use different versions of the full calibrated economy to compare the contributions of different characteristics of the model. In particular, I will add step-wise (i) heterogeneity in the frequency of price change (Case 2), (ii) in the intermediate share (Case 4), and asymmetry in the network structure (Case 5) to the multi-sector model without networks (Case 1). Eventually, Case 6 represents the fully calibrated U.S. economy in 2007.

**Case 1:** “Homogenous Multi-Sector Economy”: Multi-sector model, no production network, homogenous frequency of price adjustment

**Case 2:** “Heterogenous Multi-Sector Economy”: Multi-sector model, no production network, heterogenous frequency of price adjustment

**Case 3:** “Symmetric Input-Output Economy”: Symmetric production network, homogenous intermediate share, homogenous frequency of price adjustment

**Case 4:** “Heterogeneous Intermediate Shares Economy”: Symmetric production network, heterogeneous intermediate shares, homogenous frequency of price adjustment

**Case 5:** “Asymmetric Network Economy”: Asymmetric production network, homogenous intermediate shares, homogenous frequency of price adjustment

**Case 6:** “Full 2007 Economy”: Asymmetric production network, heterogeneous intermediate shares, heterogenous frequency of price adjustment

**Monte Carlo Evidence.** In this section I will simulate the model to calculate the implied slope components. The model does not allow for an analytical solution of the slope because of the endogenous components from relative prices. Instead the endogenous components that generate the bias will depend on the Taylor rule or the series of shocks. The system formed by the equilibrium described as in Section 3.6 and the calibration in Section 4.1 is simulated 2000 times for 200 periods, respectively. Aggregate inflation, output gap, sectoral over production and relative price gaps are collected at each repetition. Each simulation gives rise to an aggregate series for inflation and the components of marginal costs. These can then be used to estimate



an aggregate Phillips curve and the components of the slope as well as the bias. The mean estimates of the Phillips curve are then reported, averaged over 2000 repetitions. Using medians instead does not change the results.

Panel (a) of Table 7 shows the three implied slope components  $\Phi^{Std}$ ,  $\Phi^{Multi}$ ,  $\Phi^{StratComp}$  as well as their sum  $\Phi^{\hat{\pi}}$ .

The implied slope is derived from projecting the slope components on the output gap according to

$$\Phi^x = \beta^x * \hat{y}_t + \epsilon_t$$

where  $x = \{Std, Multi, StratComp, \hat{\pi}\}$  and  $\beta^{\Phi^{\hat{\pi}}}$  corresponds to the aggregate sensitivity of inflation to the output gap in this economy.

The first column of Table 7 corresponds to the multi-sector economy without production networks or intermediate goods and with homogenous frequency of price adjustment. In this economy, the slope of the Phillips curve is equal to the standard slope coefficient,  $\Phi^{Std}$ , and the econometrician is estimating exactly this coefficient without a bias.<sup>17</sup>

The multi-sector economy without intermediate goods but with heterogenous degrees of nominal rigidities is illustrated in Column two. This introduces relative price gaps as in Carlstrom et al. (2006) and a bias in the estimation of the Phillips curve due to strategic complements,  $\Phi^{StratComp}$ . This has a dampening effect on the standard slope<sup>18</sup> and the estimated slope is smaller than in case 1.

The production network is introduced step-wise in the next four columns. The network structure affects the different components of the aggregate Phillips curves in three ways. First, the size of the standard slope coefficient is reduced. The effect is stronger for heterogenous intermediate shares, indicating that sectors with large intermediate shares tend to have larger degrees of nominal rigidity. Second, the component in the Phillips curve related to the gaps in the relative size of total over production is positive which biases the aggregate estimated slope. The size of this component depends on the network multiplier and centrality of sticky sectors. Column 5 illustrates that in the asymmetric network, the centrality of sticky sectors has increased which decreases the slope coefficient relative to the symmetric network in column 3. The third effect is that the network structure affects the degree of strategic complementarity. Due to interaction effects between strategic complementarities and the network structure the component is smaller than in case 2.

In the calibrated economy in 2007, the standard slope component and the component due to strategic complementarities offset each other almost completely. As a result, the estimated slope

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<sup>17</sup>Specifically, the slope collapses to the textbook one-sector economy,  $\kappa = \kappa(\bar{\theta})$ , where  $\bar{\theta}$  is the average degree of price rigidity in the economy.

<sup>18</sup>The standard slope coefficient corresponds now to the weighted average of sectoral coefficients.



coincides with the bias introduced from the network via the over-production gap,  $\Phi^{Mult}$ .

The fourth row of Table 7 gives us the aggregate sensitivity of inflation to the output gap an econometrician would recover in this economy. Panel (b) of Table 7 confirms the previous result by presenting the slope an econometrician would estimate using simulated data on inflation and the output gap from the model. The deviation arises because the econometrician has no knowledge about expected inflation and attempts to control for this. In the presence of production networks, the slope is cut approximately in half, 0.42 vs. 0.249.

Eventually, I will look at the implications of those models for the network characteristics introduced in Section 3. First, Panel (c) of Table 7 displays the network multipliers. In the two multi-sector economies, the multiplier is one. In the production economies, heterogeneity in the intermediate shares increases the multiplier (column 4), while the actual input-output structure of the U.S. economy decreases it (column 6). This indicates that more central sectors have smaller intermediate shares.

## 6 The Slope of the Phillips curve over Time

In this section, I will calibrate the multi-sector production network model for an economy in each of the years between 1963 and 2017. For each period, I calculate the model implied sensitivity of inflation to the output gap. Then, I will decompose the aggregate into the three different components introduced in the previous section. Eventually, I analyze how structural transformation in value-added and in the production network has contributed to the flattening of the Phillips curve.

### 6.1 The Production Network Model Implied Slope

The model implied sensitivity of inflation to the output gap is flattening over time. The dashed line in Figure 12 depicts the model implied slope of the Phillips curve. This corresponds to  $\Phi^{\hat{\pi}}$  in Section 5. The line matches the shape and the timing of the identified changes to the Phillips curve. The three episodes are visible, with the first period extending until the middle of the 80's. Also we can see a slight rebound and a generally diverging behaviour of the slope after 2000. From the peak in the 80's until the beginning of the 21st century, the slope decreases by about 15%. This corresponds to 25 to 50 percent of the total decrease in the sensitivity that I have presented in Section 2. Compared to the evidence from the data in Section 1, the level of the slope is larger than data would suggest. Therefore, I want to focus on the change over time and discuss potential sources for the bias in the level, below.

## 6.2 Components

Changes to the production network are thus able to explain a significant part of the flattening of the curve. To understand the sources of this flattening, Figure 13 illustrates the paths of the different components of inflation sensitivity. The solid blue line, represents the slope coefficient associated with the network multiplier and the overproduction of the real value of production compared to value-added. It constitutes the main source of the decrease in the slope decreasing by about 50% over time. Figure 13 allows for a further comparison with changes in the standard slope (black dashed) as well as strategic complementarities (solid red). The other two lines are strongly negatively correlated, cancel each other out in most the periods, oscillating around 0.8 and -0.8, respectively. Thus, structural changes in the production network structure of the economy is the most important source of the flattening of the Phillips curve. The next two sections, analyze the roles of two important dimensions of the model and how they interacted with the changes in the network structure: (i) the frequency of price adjustment and (ii) the intermediate good share.

### 6.2.1 The Role of Reallocation of Centrality to Sticky Sectors

In Section 4, I described a the changing production structure of the U.S. economy. First, I observed an increase in the centrality of services as measured by their output shares. Second, a decrease in the network multiplier. The degree of nominal rigidity plays an important role in New Keynesian models. When the propability of being able to adjust prices decreases, the average duration of firm's prices increase and so it becomes more likely that those prices are away from their optimal level. Sectors with larger nominal rigidities (larger calvo parameters,  $\theta_k$ ) therefore have a smaller sensitivity of inflation to the output gap.

Changes to the network have decreased the aggregate degree of nominal rigidity by shifting production towards stickier sectors - i.e. sectors with a larger calvo parameter.

To give a first impression of this reallocation, Figure 14 compares the sectoral heterogeneity in nominal rigidities in 1963 and 2017. Each sector is represented by a bubble, where the size of the bubble corresponds to the centrality of the sector measured by its output share,  $\delta_k$ , in 1963 and 2017, respectively. The figure shows that service sectors (red bubbles) increase in centrality and that they display a larger rigidity. In contrast a number of formerly important manufacturing sectors (blue) become irrelevant in 2017.

To further build intuition, Figure 15 displays the average degree of nominal rigidity in the economy where the weights are ever the value-added shares (solid) or output shares (dashed line, right axis) of the respective sector. The average degree of nominal rigidity increases over time, particularly, after 1980 irrespective of the weighting. Inspecting the scales, however, the change is bigger if sectors are weighted by their output shares as in the production network model.

Finally, to give a direct estimate of the impact of this increase in the overall stickiness in this economy, I perform a counterfactual exercise. Figure 18 shows the path of the model implied inflation sensitivity assuming homogenous calvo rigidities across sectors. Figure 18 confirms the importance of the sectoral reallocation in calvo rigidities mechanism. Preventing this channel, the slope of the Phillips does not flatten over time.

### 6.2.2 Heterogeneity in Intermediate Shares and the Network Multiplier

The sensitivity of inflation to the output gap increases in the size of the network multiplier because a larger total production requires more labor input. The most important determinant for the network multiplier is the markup adjusted, output share weighted average intermediate input share,  $\gamma$ . On the other hand, an decrease in the intermediate good share decreases the importance for relative input price gaps for marginal costs, strategic complementarities.

In the calibrated model, I abstract from heterogeneity in markups. Hence, the network multiplier is proportional to the average output share weighted intermediate share. Panel A of Figure 17 shows the strong connection between the two. The network multiplier falls in the 1980's and then increase again the mid 2000's.<sup>19</sup> The underlying reason for this is twofold and can be explained by observing the cross-section.

Figure 14 shows the sectoral change in the intermediate shares between 1963 and 2017. First, manufacturing firms (blue) tend to have larger intermediate good shares than service sectors (red). The change in the network in the 80's therefore decreased the aggregate intermediate good share and similarly the network multiplier. In this way, the decrease in the network multiplier contributes to the flattening of the Phillips curve. Second, Figure 14 furthermore shows that the intermediate good share of service sectors (red) has increased from 1963 to 2017. This increase took place at the beginning of the 21st century and resulted in an increase in the aggregate intermediate good share. The increase of services' intermediate good share is the cause of the increase in the implied slope of the model in the 2000's in Figure 12 via the network multiplier.

Eventually, Figure 18 performs a similar counterfactual exercise as in the last section. In particular, Figure 18 displays the implied inflation sensitivity of the model under an homogenous intermediate goods share. The flattening appears to be stronger than in the baseline case. This shows that the two opposing mechanisms of changes in the intermediate share offset each other. In fact, it appears that the change in intermediate shares decreases the role of strategic complementarities more than it dampens the slope of the Phillips curve by decreasing the network multiplier.<sup>20</sup>

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<sup>19</sup>Panel B of Figure 17 also provides an external validity exercise. Despite not being targeting, the model does a good job at matching the observed network multiplier from the data.

<sup>20</sup>In the calibration exercise, the decrease in the network multiplier is only caused by a change in the intermediate share since we abstract from changes in markups. A number of recent studies has shown, however, that market power may have risen in a number of sectors in the economy. Increases in markups would be an alternative

### 6.2.3 The Role of Structural Transformation

Finally, I want to look at the role played by structural transformation, i.e. changes in the value-added shares of sectors. Therefore, we first compare the changes in centrality to those changes in the sectoral GDP shares as measured by final demand from the BEA Use table. The Figure 7 shows the change in final demand for each of the 53 sectors from 1963 to 2017. While the construction, retail and most Manufacturing sectors (Codes 3) see a decline in the importance, services (5-8) increase strongly in GDP senses. The increase is strongest for services related to health care such as “Ambulatory health care services” (621) or “Hospitals and nursing and residential care facilities” (622). In contrast, the food and textile productions have seen a large decrease in their importance.

Next, I will investigate the impact of this transformation on the slope. Figure 18 shows the result of a counterfactual exercise where the value-added shares,  $V_C$ , are hold constant at their levels from 1963. Any change in the solid line is due to structural changes in the network structure as opposed to changes in the value-added shares. Figure 18 shows that structural transformation in networks and in value-added shares each contribute half to the total decline in the slope. The underlying reason is that structural transformation has also led to the reallocation towards sectors that have larger rigidities. Figure 15 shows, however, that the effect is stronger when also taking into account the network structure. In this sense, in order to understand the flattening of the Phillips curve it is important to consider structural transformation in value-added shares and the network structure together.

## 7 Conclusion

A growing literature documents that the Phillips curve has flattened over time. I contribute to this discussion by providing evidence that changes to the network structure can be an important explanation. Using a multi-sector model with production networks, I show that input-output linkages in the production function of firms affect inflation dynamics. Moreover, I identify the relationship between different network characteristics and the sensitivity of inflation to the output gap. When calibrated to the U.S. economy from 1963-2017, changes to the network structure of the economy are able to explain a significant part of the flattening of the Phillips curve. In this project, I abstracted from two dimensions of change in the network that are promising avenues for future research: (i) international linkages in production networks and (ii) the role of rising market power for the network multiplier. Accounting for production networks and its changes over time has important implications for inflation dynamics and contributes to better understand the changes to the Phillips curve relationship in the past decades.

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explanation in the observed decrease in the network multiplier with an unambiguous dampening effect on the slope.

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## Tables and Figures

Specification	Break Date	p-value
$\pi^{PCE}, y_t$	1982q3	0.0323
$\pi^{CPI}, y_t$	1983q2	0.0032
$\pi^{PCE}, u_t$	1981q1	0.0000
$\pi^{PCE}, ugap_t$	1982q3	0.1260
$\pi^{CPI}, u_t$	1995q3	0.0008
$\pi^{CPI}, ugap_t$	1983q2	0.0917
IV: $\pi^{PCE}, y_t$	1983q2	0.0502
IV: $\pi^{PCE}, ugap_t$	1982q1	0.0942
IV: $\pi^{PCE}, ur_t$	1982q1	0.0009

Table 1: This table displays the results of an Andrew's test (1993) with 15% trimming and supremum LR-test (Quandt, 1960).

Parameter	Description	Value
$\sigma$	Constant relative risk aversion	1
$\nu$	Inverse of Frisch elasticity of labor	1
$\epsilon$	Constant elasticity of substitution	6
$\phi_\pi$	Inflationary response of the Taylor Rule	1.5
$\phi_c$	Output-gap response of the Taylor Rule	0.5/4

Table 2: This table displays the calibration of parameters.

Top 10 central industries in 1963		Top 10 central industries in 2017	
23	Construction	531	Real estate
311FT	Food, beverage, tobacco products	5412OP	Manage Prof scientific technical
44RT	Retail trade	42	Wholesale trade
531	Real estate	81	Other services, except government
81	Other services, except government	44RT	Retail trade
5412OP	Manage Prof scientific technical	622	Hospitals
42	Wholesale trade	23	Construction
3361MV	Motor vehicles, and parts	524	Insurance and related
111CA	Farms	621	Ambulatory health care services
325	Chemical products	311FT	Food, beverage, tobacco products
Bottom 10 central industries in 1963		Bottom 10 central industries in 2017	
514	Information services	486	Pipeline transportation
486	Pipeline transportation	315AL	Apparel and leather products
493	Warehousing and storage	483	Water transportation
562	Waste management services	323	Printing and support activities
213	Support activities for mining	337	Furniture and related products
483	Water transportation	313TT	Textile mills
523	Securities	482	Rail transportation
721	Accommodation	212	Mining, except oil and gas
323	Printing and support activities	113FF	Forestry, fishing, and related
481	Air transportation	562	Waste management

Table 3: This table compares sectors' centrality in 1963 vis-a-vis 2017. Bureau of Economic Analysis and author's calculations.



Top 10 rank improvements from 1963 to 2017							
	Industry	$\theta_k$	$\gamma_k^{1963}$	$\gamma_k^{2017}$	$\Delta W_c$	$W'_k,1963$	$W'_k,2017$
622	Hospitals	0.79	0.24	0.45	0.05	0	0.01
531	Real estate	0.50	0.13	0.31	0.01	1.61	3.26
5412OP	Manage Prof scientific technical	0.78	0.37	0.37	0.00	3.46	6.17
621	Ambulatory health care services	0.83	0.39	0.34	0.03	0.01	0.07
561	Administrative and support services	0.65	0.22	0.40	0.00	0.64	2.96
524	Insurance carriers and related activities	0.82	0.49	0.57	0.01	1.53	1.58
5411	Legal services	0.87	0.45	0.27	0.02	1.05	1.46
523	Securities	0.55	0.53	0.47	0.01	0.22	0.99
42	Wholesale trade	0.45	0.16	0.38	0.01	2.42	3.70
521CI	Federal Reserve banks	0.08	0.37	0.31	0.01	1.75	1.53
Top 10 rank decline from 1963 to 2017							
	Industry	$\theta_k$	$\gamma_k^{1963}$	$\gamma_k^{2017}$	$\Delta W_c$	$W'_k,1963$	$W'_k,2017$
311FT	Food, beverage, tobacco products	0.61	0.71	0.72	-0.04	1.63	0.85
23	Construction	0.42	0.59	0.49	-0.05	0.87	0.56
44RT	Retail trade	0.56	0.41	0.39	-0.04	0.69	0.46
111CA	Farms	0.02	0.52	0.65	-0.01	1.15	0.68
3361MV	Motor vehicles and parts	0.69	0.64	0.78	-0.02	1.08	0.81
315AL	Apparel	0.81	0.69	0.64	-0.02	0.5	0.35
331	Primary metals	0.20	0.61	0.72	0.00	2.64	1.64
333	Machinery	0.78	0.46	0.60	-0.01	0.96	0.85
313TT	Textile mills	0.75	0.70	0.66	0.00	1.32	0.59
332	Fabricated metal products	0.73	0.54	0.58	0.00	1.76	1.62

Table 4: This table compares the change in sectors' centrality from 1963 to 2017 and report some key characteristics such as frequency of price change  $\theta$ , intermediate share  $\gamma$ , change in value added share  $\Delta W_c$ , and outdegree  $W'$ . Bureau of Economic Analysis and author's calculations.

	Multi-Sector		Production Network			2007 Economy
	Homogenous Frequency	Heterogenous Frequency	Symmetric Network	Heterogenous Int. Shares	Asymmetric Network	
Panel (a) Slopes						
$\beta^{Std}$	0.48	2.63	0.94	0.83	0.94	0.83
$\beta^{Mult}$	0	0	0.33	0.28	0.25	0.22
$\beta^{StratComp}$	0	-2.21	-1.04	-0.88	-0.94	-0.81
$\beta_{\hat{\pi}}$	0.48	0.42	0.23	0.225	0.246	0.249
Panel (b) Estimated Slopes						
$\beta^{OLS}$	0.51	0.448	0.243	0.237	0.26	0.263
Panel (c) Network Statistics						
$\Phi^{NM}$	1.00	1.00	1.649	1.680	1.649	1.663

Table 5: This table compares different calibrated economies of the model. Panel (a) shows the implied slopes of the model, which is the regression coefficients from projections on simulated data of the model:  $\Phi^x = \beta^x * \hat{y}_t + \epsilon_t$ . Panel (b) shows the result of projecting simulated data for the output gap on simulated data for inflation. Panel (c) displays the network multiplier in this economy. The first two columns represent multi-sector models without intermediate goods and homogenous or heterogeneous degrees of nominal rigidity respectively. Columns 3 and 5 consider the production network model with homogenous intermediate shares but a symmetric network or the actual (asymmetric) network. Column 4 adds heterogeneity in intermediate good shares to the symmetric network in column 3. Column 6 represents the full production network model calibrated to 2007.

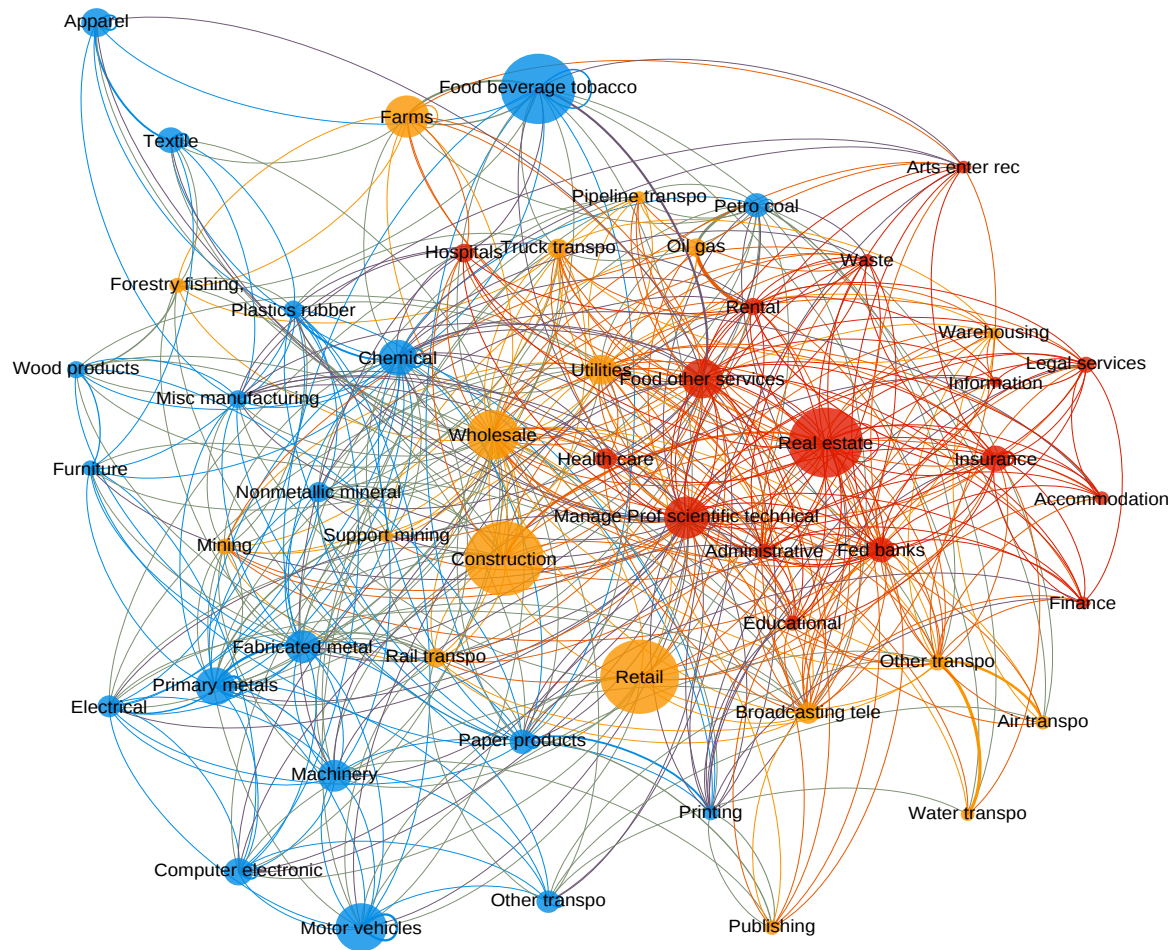


Figure 1: This figure displays the production network corresponding to US Input-Output data in 1963. Each node in the network corresponds to a sector in the 1963 input-output data, while each edge corresponds to an input-supply relation between two sectors. Larger nodes represent more central sectors in terms of output shares. Color-codes represent: (i) Manufacturing (blue), (ii) services (red) and (iii) others (orange). Source: Bureau of Economic Analysis. The Figure is drawn with the software package Gephi.

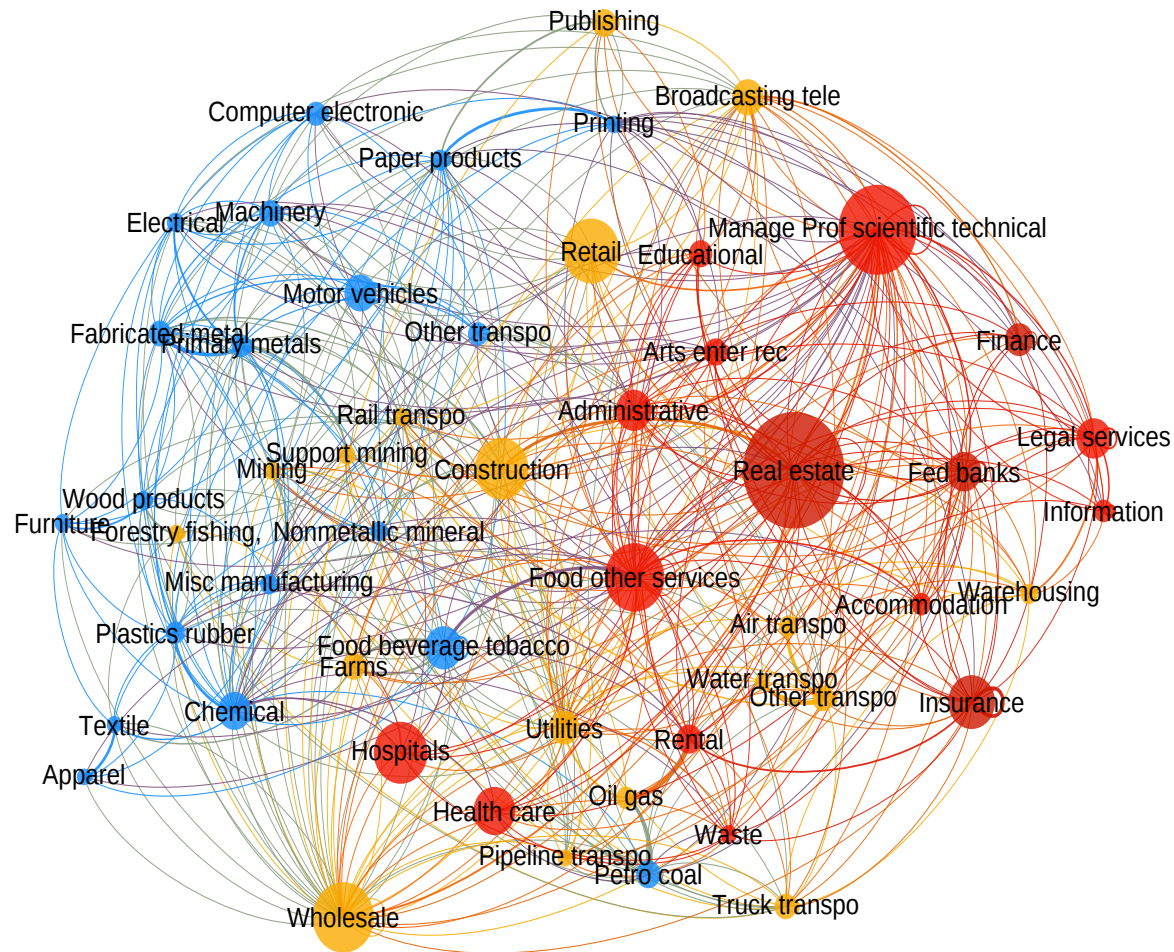


Figure 2: This figure displays the production network corresponding to US Input-Output data in 2017. Each node in the network corresponds to a sector in the 2017 input-output data, while each edge corresponds to an input-supply relation between two sectors. Larger nodes represent more central sectors in terms of output shares. Color-codes represent: (i) Manufacturing (blue), (ii) services (red) and (iii) others (orange). Source: Bureau of Economic Analysis. The Figure is drawn with the software package Gephi.

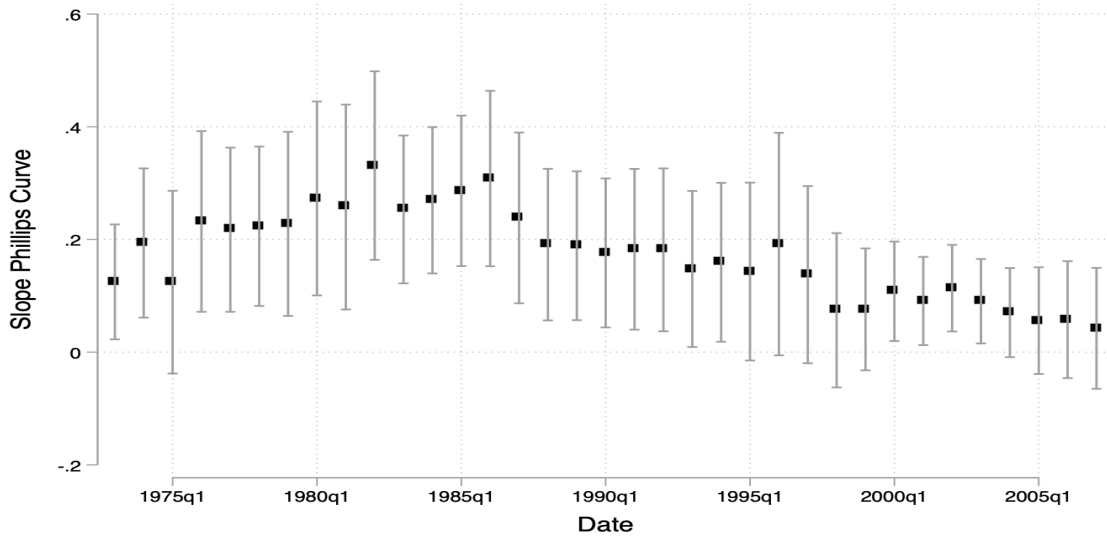


Figure 3: This figure displays the results of a rolling window estimates of the Phillips curve as in Equation (1). Window size is 50 quarters.

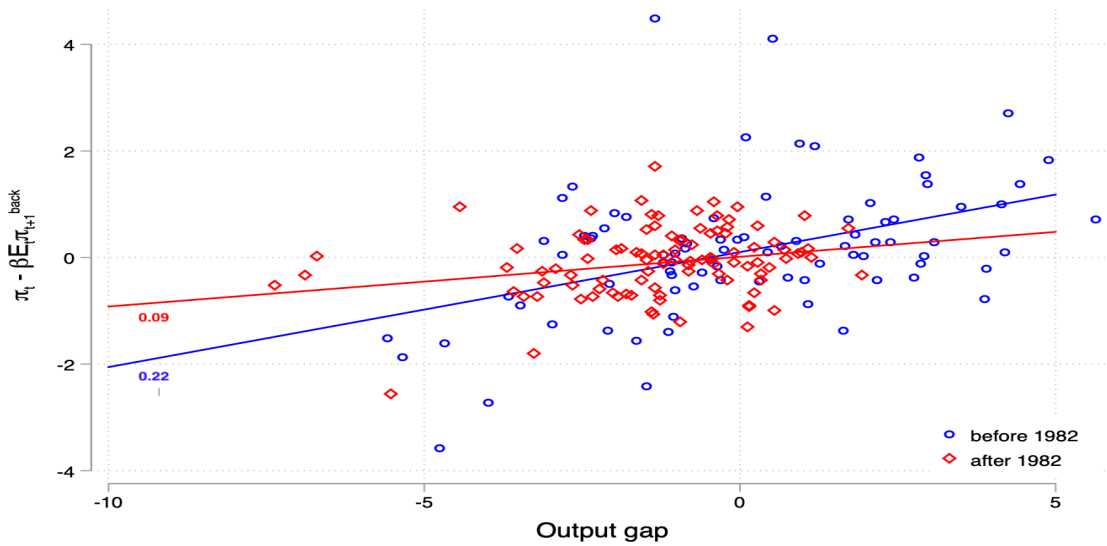


Figure 4: This figure illustrates the flattening of the Phillips curve. It displays observations for predicted inflation and the output gap before and after 1985Q1 together with the implied slope estimates.

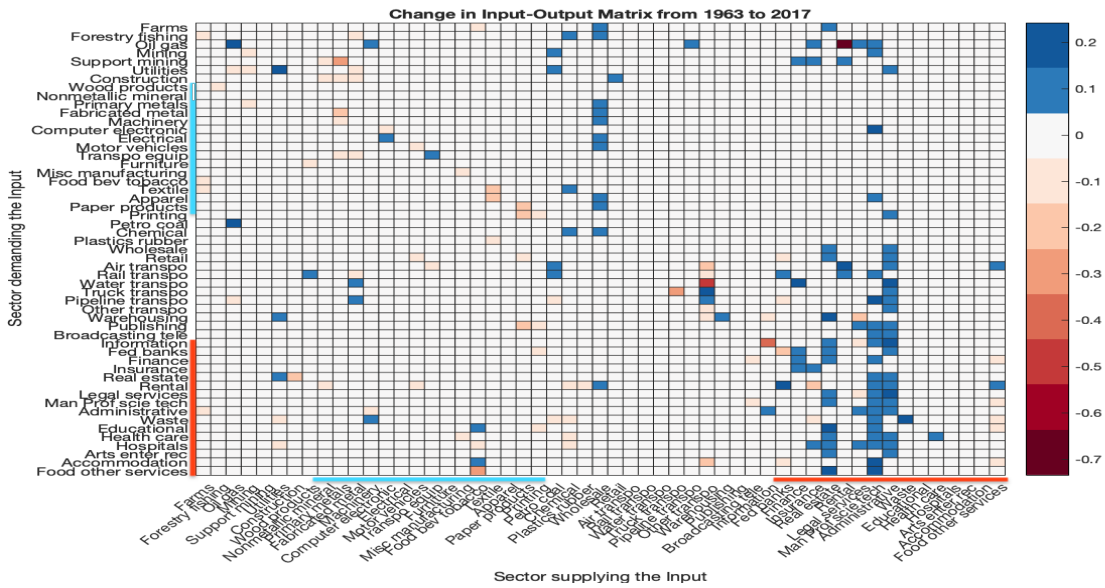


Figure 5: This figure shows the change in the Input-output matrix from 1963 to 2017 as created by the guidelines of BEA handbook. While increases in the intensity of demands or provisions are colored blue, decreases in the importance of an edge is colored in red.

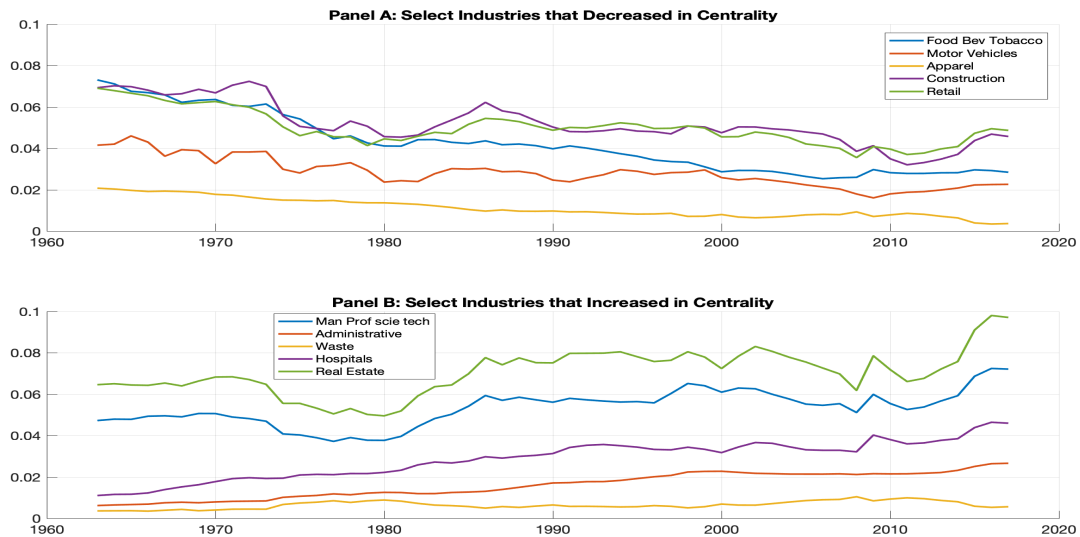


Figure 6: This figure displays the change in the output shares from 1963 to 2017 for selected industries. Panel A shows industries that were among the top decreases in centrality, while Panel B displays industries that increased most in centrality. Sources: Bureau of Economic Analysis and authors calculations.

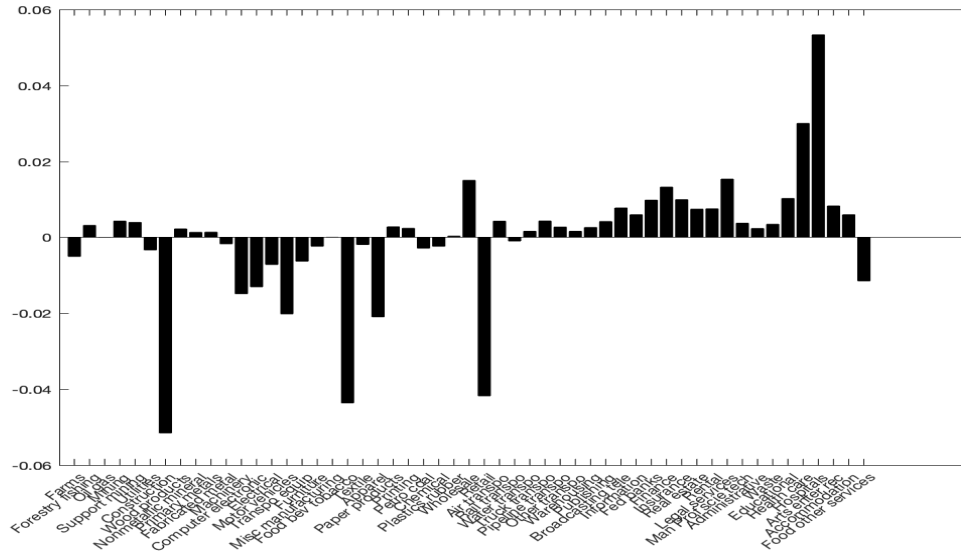


Figure 7: This figure illustrates the change in sectoral value added share as measured by final demand from the BEA Use table from 1963 to 2017. While Manufacturing sectors (Codes 3) see a decline in the importance, services (5-8) increase strongly in GDP senses.

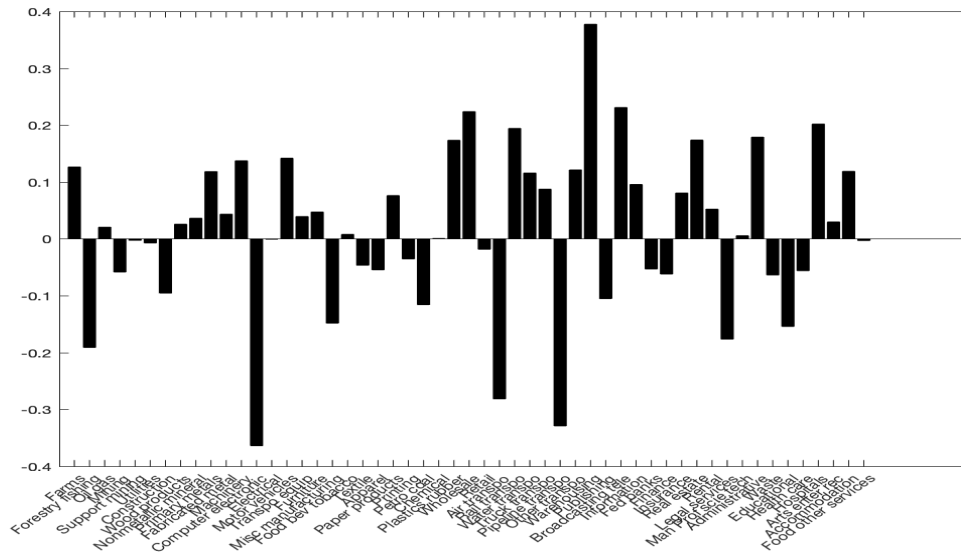


Figure 8: This figure shows the change in sectoral intermediate good use share as measured by the share of intermediates vis-a-vis value added in industries' production from the BEA Use table from 1963 to 2017. The pattern is not as clear as with the GDP share, but generally Manufacturing sectors do not see a strong increase in in intermediate use as Services (5-8).



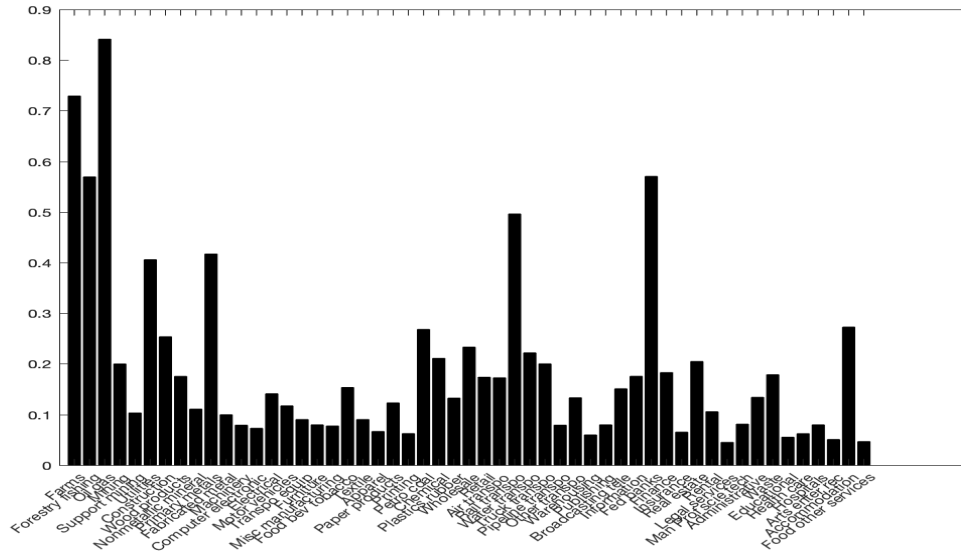


Figure 9: This figure plots the distribution of frequency of price change for each sector. It is measured as the average proportion of goods within each sector from the BEA survey of PPI firms that changes prices. I thank Michael Weber for providing the data. The pattern is not as clear as with the GDP share, but generally Manufacturing sectors tend to have a higher frequency of price change, i.e. they are less rigid than Services (5-8).

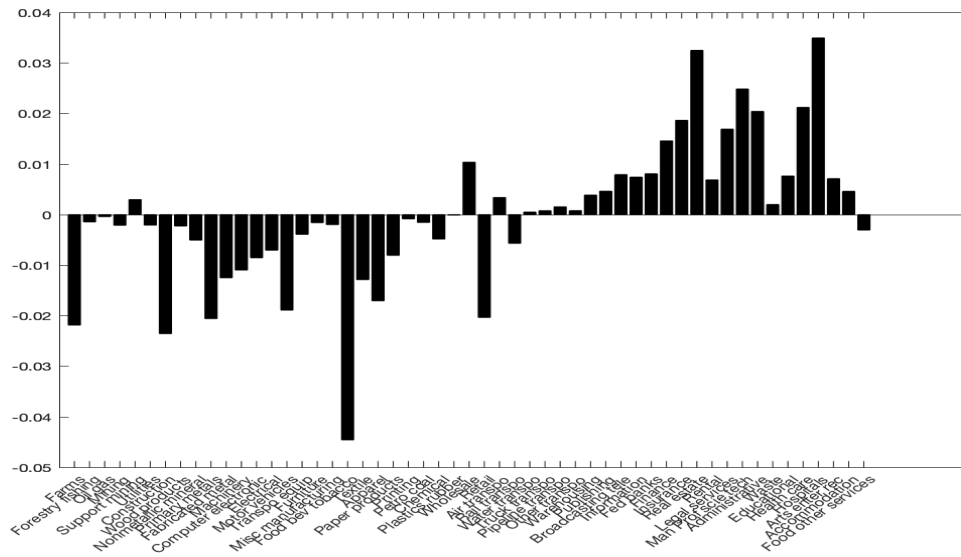


Figure 10: This figure shows the change in sectoral output shares, a measure of the network centrality of a sector. It is apparent how services became the most central sector in the economy.



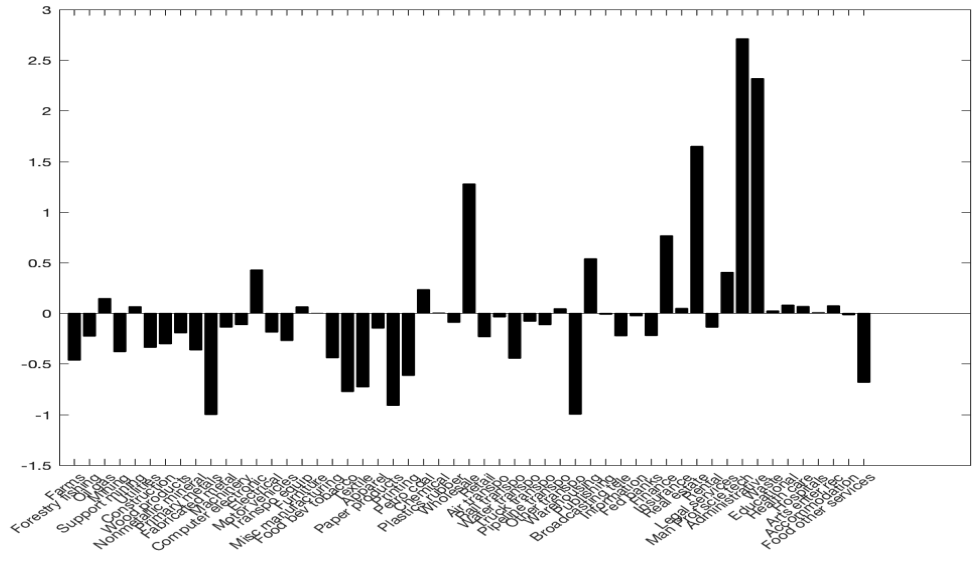


Figure 11: This figure displays the change in sectoral outdegrees, which are another measure of network centrality and equal to  $W'$ . Services became the most central sector in the economy also according to the outdegree.

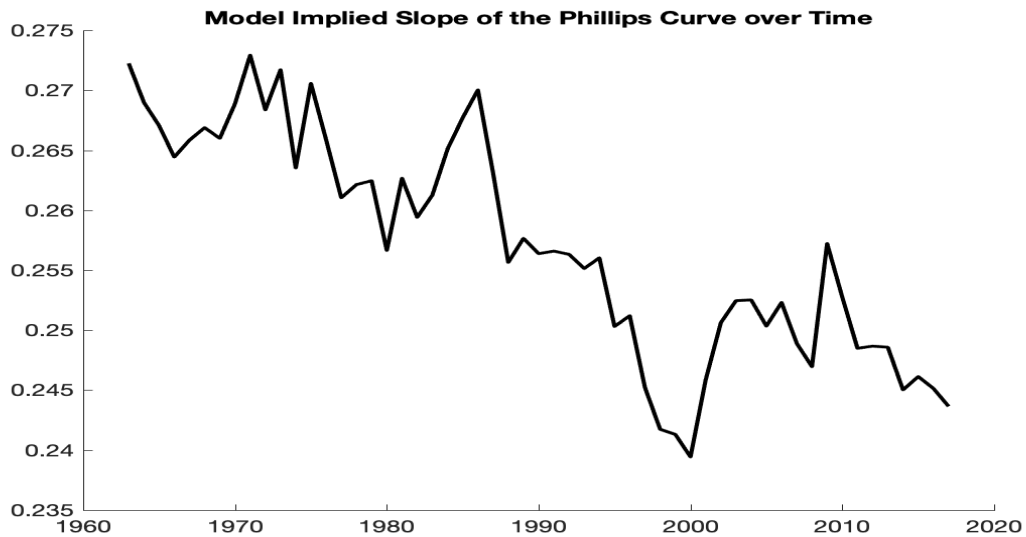


Figure 12: This figure shows the change in the model implied slope over time. The model is calibrated to each year from 1963 to 2017 and the implied sensitivity of inflation with respect to the output gap calculated. Additionally, the figure shows the linear projection of simulated data for the output gap on inflation.

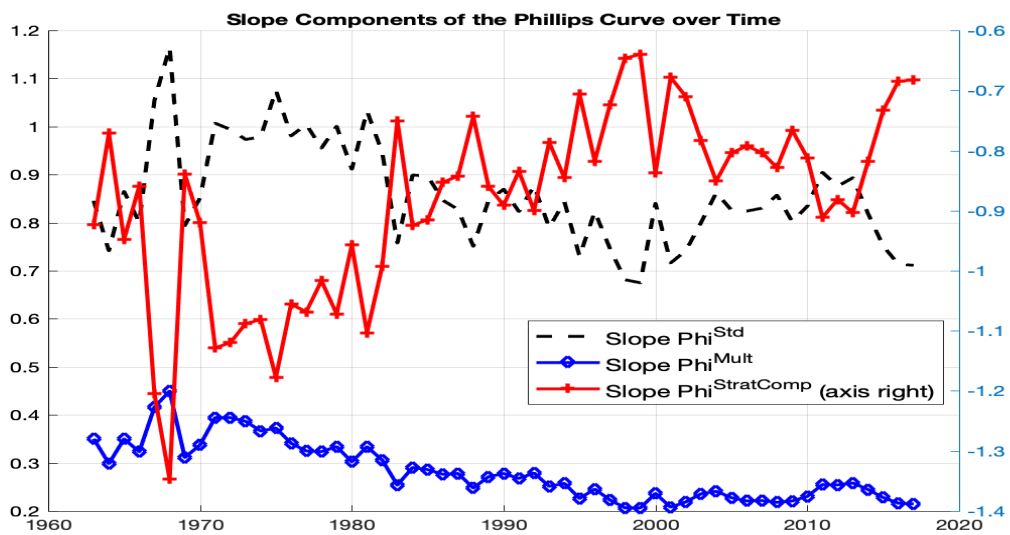


Figure 13: This figure decomposes the change in the model implied slope over time. The model is calibrated to each year from 1963 to 2017 and the implied sensitivity of inflation with respect to the output gap calculated. The figure shows the three components of inflation dynamics: (i) the standard slope (black, dashed), (ii) the multiplier slope (blue solid) and (iii) the strategic complementarity slope (red, solid).

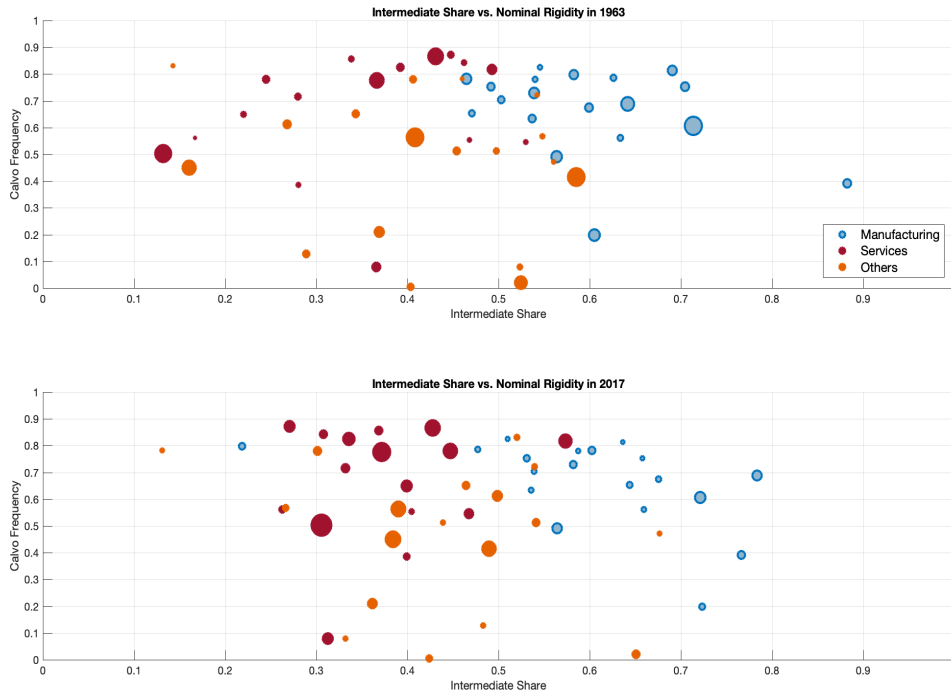


Figure 14: This figure shows the sectoral heterogeneity in (i) the degree of nominal rigidity and (ii) intermediate goods shares and the change from 1963 to 2017. Each bubble represents a sectors and the size and the size corresponds to the centrality of that sector as measured by the output gap,  $\delta_k$ .

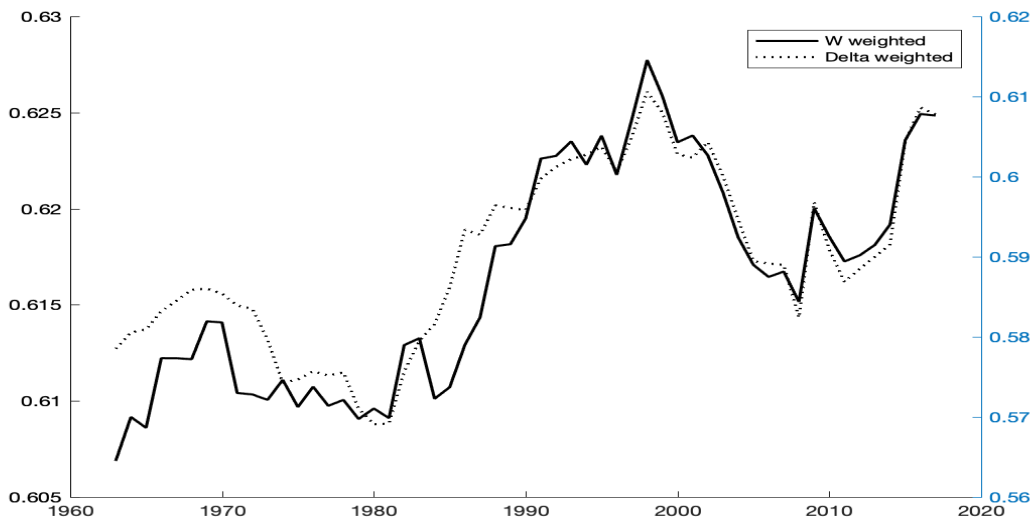


Figure 15: This figure shows the evolution of the average degree of nominal rigidity in the economy. While the solid line weights each sector by its value-added share, the dashed line weights by the output share.

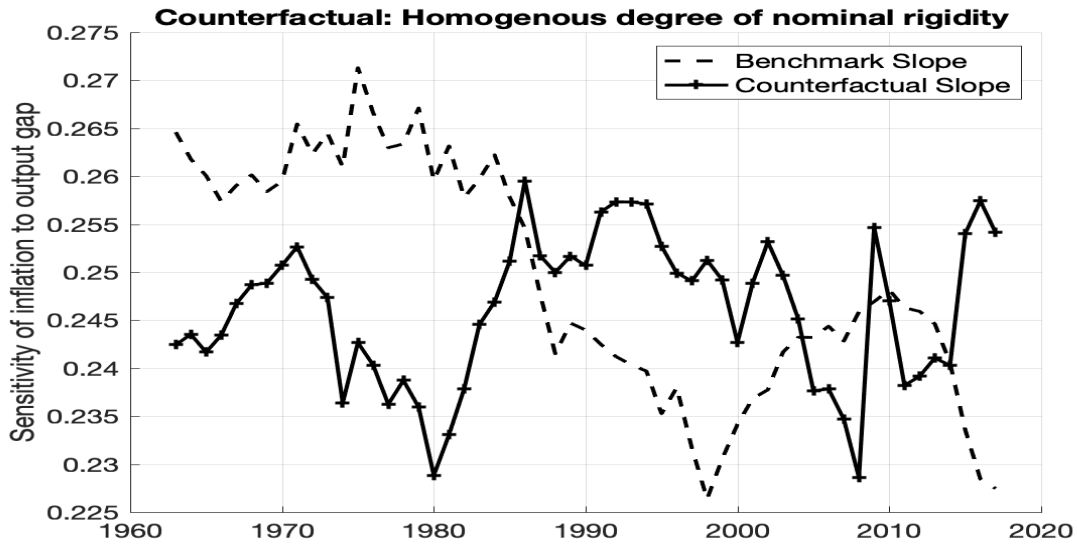


Figure 16: This figure displays the counterfactual evolution of the implied slope of the Phillips curve under an homogenous degree of nominal rigidities across sectors.

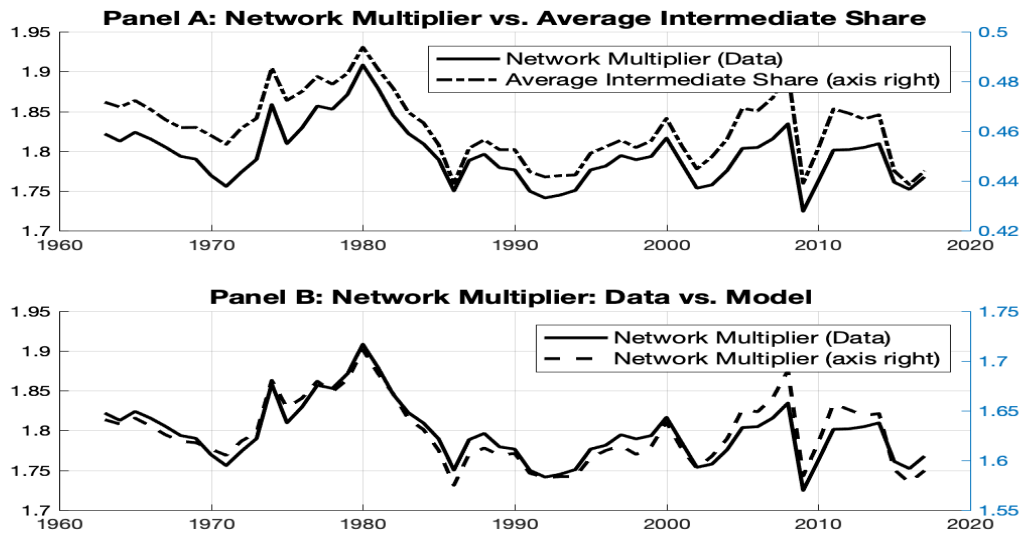


Figure 17: This figure compares the network multiplier with the model implied network multiplier (Panel A) and to the average intermediate good share in the economy (Panel B).

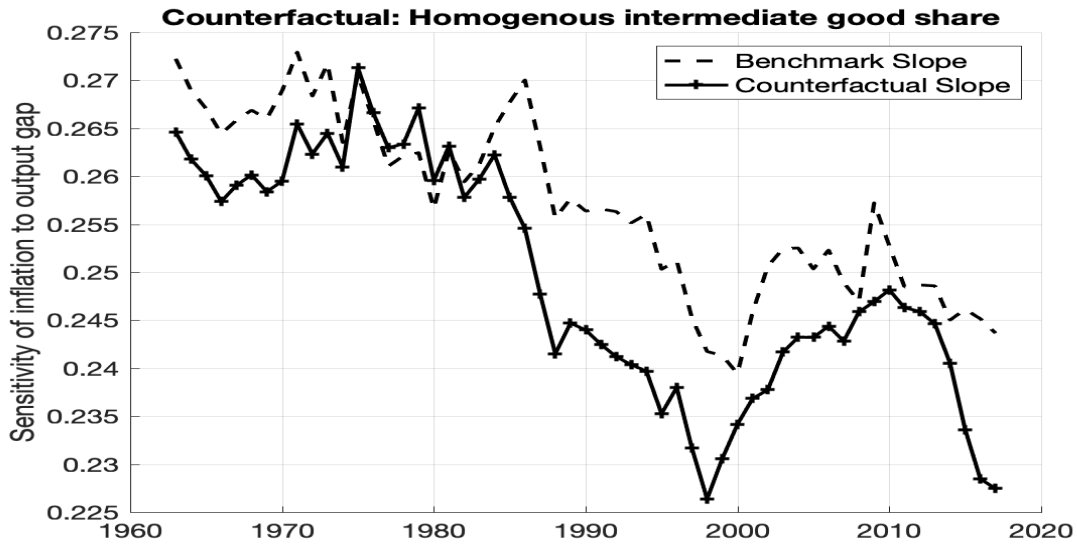


Figure 18: This figure displays the counterfactual evolution of the implied slope of the Phillips curve under an homogenous intermediate good share across sectors.

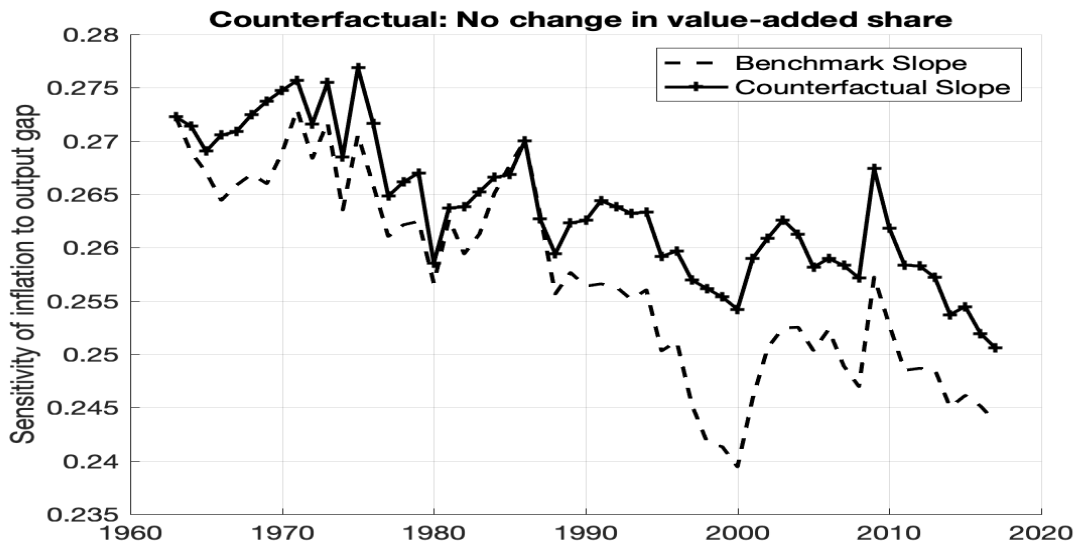


Figure 19: This figure illustrates the counterfactual evolution of the implied slope of the Phillips curve when sectoral value added shares were assumed to remain at their value from 1963.